Exam Advice

First Class Honours 70%

Pass mark 40%

Exam has six questions.

- $2\frac{1}{2}$ correct questions gives a pass.
- Or 7 marks (out of 17) per question over 6 questions.

CA counts 40% – Must pass

Exams count 60%

CA Sem I Counts 20%

Exam Sem I counts 30%.

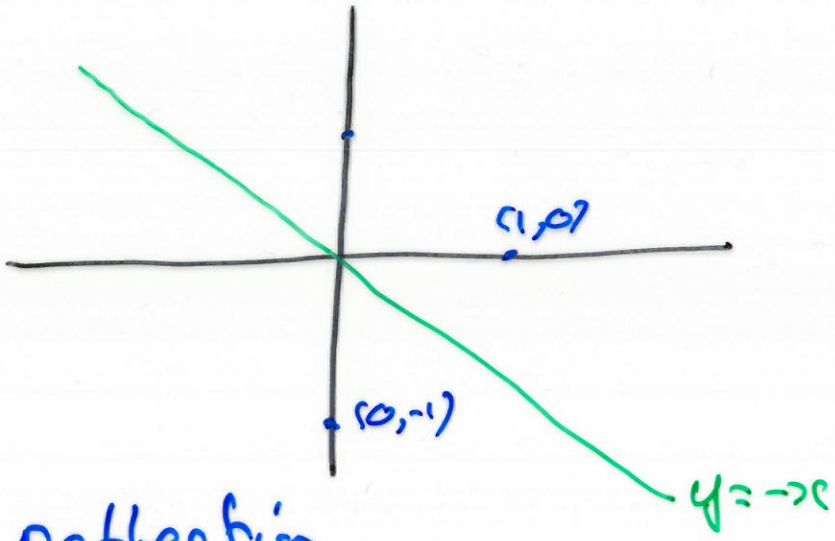
CA Sem I

48 correct questions

gives 100 %

MA180 Exam 2015-16

3a)



reflection

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}_{F} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{G} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

The matrix of $g \circ f : \mathbb{R}^2 \to \mathbb{R}^2$ is

$$GF = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

3b) (i)

$$\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

A v

so $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigen. vector with eigenvalue $\lambda = 2$

$$\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$$

So $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ is an eigen. vector with eigenvalue $\lambda = -2$.

$$D = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 1 \\ 1 & -3 \end{pmatrix}$$

MA180 Paper I, 2015-16

Q1a)

$37 = 1.20 + 17$

$20 = 1.17 + 3$

$17 = 5.3 + 2$

$3 = 1.2 + 1 = \gcd(37, 20)$

$1 = 3 - 1.2$

$= 3 - 1.(17 - 5.3)$

$= 6.3 - 1.17$

$= 6.(20 - 1.17) - 1.17$

$= 6.20 - 7.17$

$= 6.20 - 7(37 - 1.20)$

$= 13.20 - 7.37$

$\equiv 13.20 \mod 37$

Hence $20^{-1} \equiv 13 \mod 37$

x = Second digit.

Need

$$0 \equiv 0.1 + x.2 + 4.3 + 0.4$$

$$+ 7.5 + 9.6 + 6.7 + 1.8$$

$$+ 4.9 + 6.10 \pmod{11}$$

$$0 \equiv 2x + 1 + 2 - 1 - 2 + 8$$

$$+ 3 + 5$$

$$0 \equiv 2x + 5$$

$$2x \equiv -5 \equiv 6$$

$$\boxed{x \equiv 3}$$

c) i) 756

$= 2 \cdot 378$

$= 2^2 \cdot 189$

$= 2^2 \cdot 3 \cdot 63$

$= 2^2 \cdot 3^2 \cdot 21$

$= 2^2 \cdot 3^3 \cdot 7$

$\phi(p^n) = p^n - p^{n-1}$

ii) $\phi(756)$

$= \phi(2^2 \cdot 3^3 \cdot 7)$

$= \phi(2^2)\,\phi(3^3)\,\phi(7)$

$= (2^2 - 2)(3^3 - 3^2)(7 - 1)$

$= 2 \cdot 18 \cdot 6$

$= 216$

iii) $a^{\phi(m)} \equiv 1 \mod m$

$$A^{-1} = \begin{pmatrix} -3 & -2 & 2 \\ 4 & 5 & -4 \\ 2 & 1 & -1 \end{pmatrix}$$

Q2a) i)

$$A^{-1} = 3^{-1} \begin{pmatrix} 4 & -5 \\ -1 & 2 \end{pmatrix} \mod 26$$

Note: $3^{-1} \equiv 9 \mod 26$

$$A^{-1} = \begin{pmatrix} 10 & -19 \\ -9 & 18 \end{pmatrix}$$

ii) $f_D : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 10 & -19 \\ -9 & 18 \end{pmatrix} \begin{pmatrix} x-15 \\ y-20 \end{pmatrix}$

$$= \begin{pmatrix} 10 & 7 \\ 17 & 18 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 10 & 7 \\ 17 & 18 \end{pmatrix} \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

= etc.

$5^{218} \equiv 5^{216} \cdot 5^2 \equiv 25 \pmod{756}$

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2 (b)

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 4 & 1 & 4 & 0 & 1 & 0 \\ 6 & 1 & 7 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ \longrightarrow \\ R_3 \to R_3 - 6R_1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -4 & -4 & 1 & 0 \\ 0 & 1 & -5 & -6 & 0 & 1 \end{array}\right) \quad \begin{array}{l} R_3 \to R_3 - R_2 \\ \longrightarrow \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -4 & -4 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 1 \end{array}\right) \quad \begin{array}{l} R_3 \to -R_3 \\ \longrightarrow \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -4 & -4 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 + 4R_3 \\ \longrightarrow \\ R_1 \to R_1 - 2R_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -2 & 2 \\ 0 & 1 & 0 & 4 & 5 & -4 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right)$$