Yesterday

$2^{-1} \equiv 4 \mod 7$

Since $2 \cdot 4 \equiv 1 \mod 7$

$3^{-1} \equiv 5 \mod 7$

$4^{-1} \equiv 2 \mod 7$

$5^{-1} \equiv 3 \mod 7$

$6^{-1} \equiv 6 \mod 7$

$1^{-1} \equiv 1 \mod 7$

How do we find the inverse of, say, 15 mod 26?

i.e. how do we find a number k such that

$$15 \times k \equiv 1 \mod 26 \ ?$$

Answer:

Step 1: use the Euclidean algorithm to find $\gcd(15, 26) = 1$

Step 2: use the output of the Euclidean algorithm to find $15^{-1} \mod 26$.

Euclidean Algorithm

$26 = 1 \times 15 + 11$

$15 = 1 \times 11 + 4$

$11 = 2 \times 4 + 3$

$4 = 1 \times 3 + 1 = \gcd(26, 15)$

$3 = 3 \times 1 + 0 \leftarrow$ stop

$1 = 4 - (1 \times 3)$

$= 4 - 1 \times (11 - 2 \times 4)$

$= 3 \times 4 - 11$

$= 3 \times (15 - 11) - 11$

$= 3 \times 15 - 4 \times 11$

$= 3 \times 15 - 4(26 - 15)$

$= 7 \times 15 - 4 \times 26$

$\equiv 7 \times 15 \mod 26$

Hence

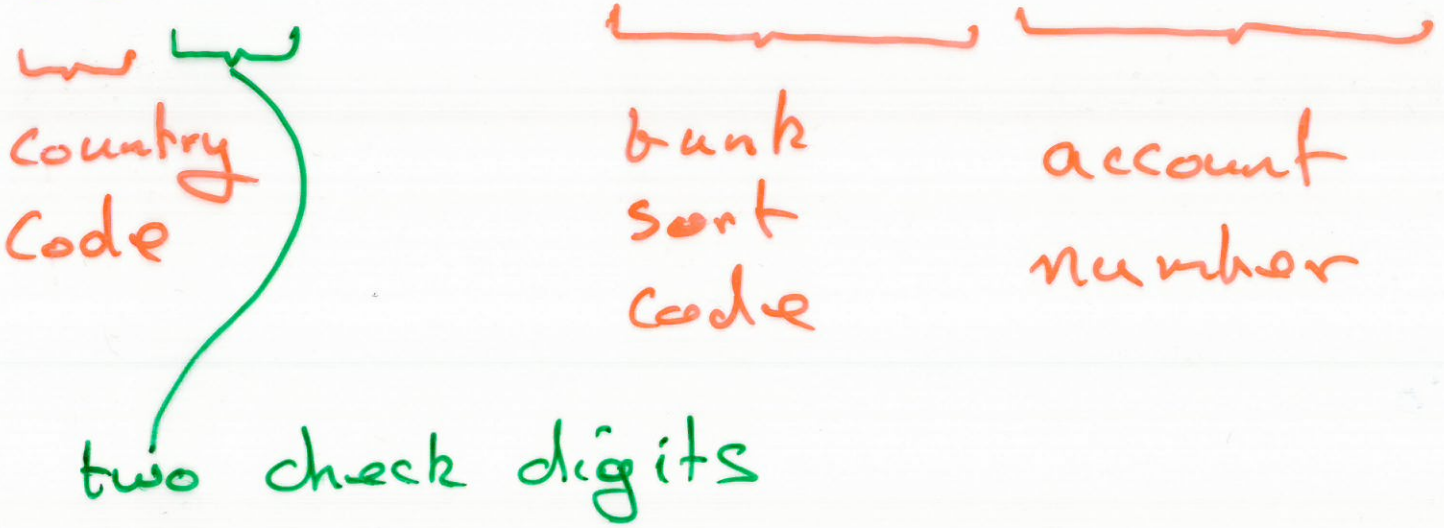
$$15^{-1} \equiv 7 \mod 26$$

(since $15 \times 7 \equiv 1 \mod 26$).

Second Application

IBAN number :

GB 82 WEST 123456 98765432



Three steps to validate an IBAN.

1) Rearrange

WEST 123456 98765432 GB 82

2) Convert letters to numbers:

A ~ 10, B ~ 11, C ~ 12, ..., Z ~ 35

32 14 28 29 1 2 3 4 5 6 9 8 7 6 5 4 3 2 16 11 8 2

3) Calculate this long number mod 97. The number must be 1 mod 97 if the IBAN is valid.

Question: How do we calculate large numbers mod 97 ?

Example Calculate

$$4321 \mod 97$$

Sol^{n}

$$4321 = 4\times 1000 + 3\times 100 + 2\times 10 + 1$$

$$\equiv 4\times 10\times 3 + 3\times 3 + 21 \mod 97$$

$$\equiv 23 + 9 + 21$$

$$\equiv 53 \mod 97$$