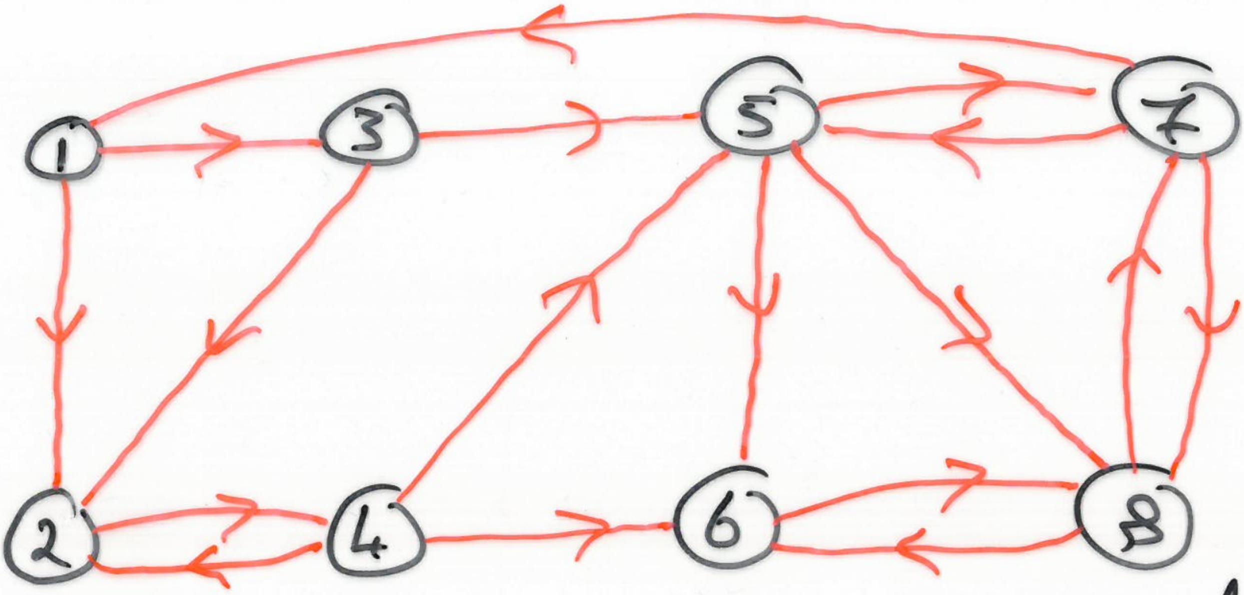
Google

A list of key words

matrices, eigenvectors, breeding rabbits, belly-buttons

results in a few web pages being listed as most likely of interest.

The WWW pages containing the key words can be represented as a diagram of nodes (one node for each page) and arrows (corresponding to a link from one page to another).



When listing pages Google first assigns a number I_n to each page P_n. I_n is the "importance" of page P_n.

$$I_1 = \frac{I_7}{3}$$

$$I_2 = \frac{I_1}{2} + \frac{I_3}{2} + \frac{I_4}{3}$$

$$I_3 = \frac{I_1}{2}$$

$$I_4 = I_2$$

$$I_5 = \frac{I_3}{2} + \frac{I_4}{3} + \frac{I_7}{3}$$

$$I_6 = \frac{I_4}{3} + \frac{I_5}{3} + \frac{I_8}{2}$$

$$I_7 = \frac{I_5}{3} + \frac{I_8}{2}$$

$$I_8 = \frac{I_5}{3} + I_6 + \frac{I_7}{3}$$

But how do we determine the numbers I_n ?

$$
\underbrace{\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{3} & 1 & \frac{1}{3} & 0
\end{pmatrix}}_{A}
\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{pmatrix}}_{v}
=
\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{pmatrix}}_{v}
$$

Note: v is an eigenvector of A with corresponding eigenvalue $\lambda = 1$.

An eigenvector for A is:

$$
v = \begin{pmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{pmatrix}
$$

Google lists the 8 pages in the order

P_8
P_6
P_7
P_5
P_2
P_4
P_1
P_3

Let A be a 2×2 matrix

<u>Defn</u> The polynomial

$$P_A(\lambda) = \det(A - \lambda I)$$

is called the <u>characteristic polynomial</u> of A.

<u>Example</u> $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$$P_A(\lambda) = \det\left(\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$= \det \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$$

$$= (2-\lambda)(2-\lambda) - 1.1$$

$$= \lambda^2 - 4\lambda + 4 - 1$$

$$= \lambda^2 - 4\lambda + 3.$$

$P_A(\lambda) = \lambda^2 - 4\lambda + 3$

$P_A(2) = 2^2 - 4.2 + 3 = -1$

$P_A(A) = A^2 - 4A + 3I$

$$= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - 4\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} 8 & 4 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

<u>Cayley-Hamilton Theorem</u>

For any 2×2 (or $n\times n$) matrix A we have

$$P_A(A) = 0I.$$