Recall

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det(A) = ad - bc$$

$$|A| = ad - bc$$

Example

$$A = \begin{pmatrix} -2 & 6 \\ 3 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & -1 \\ -3 & 7 \end{pmatrix}$$

$$AB = \begin{pmatrix} -28 & 44 \\ 3 & 25 \end{pmatrix}$$

$$|AB| = (-28)(25) - (3)(44) = -832$$

$$|A| = (-2)4 - 3.6 = -26$$

$$|B| = 5.7 - (-3)(-1) = 33$$

$$|A||B| = (-26)(32) = -832$$

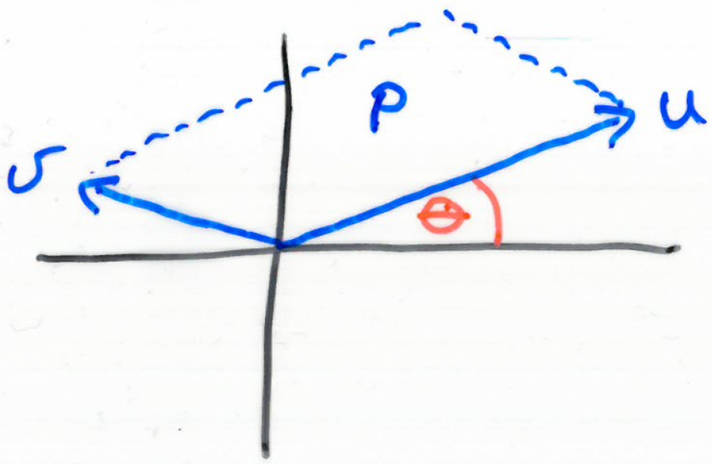
We See

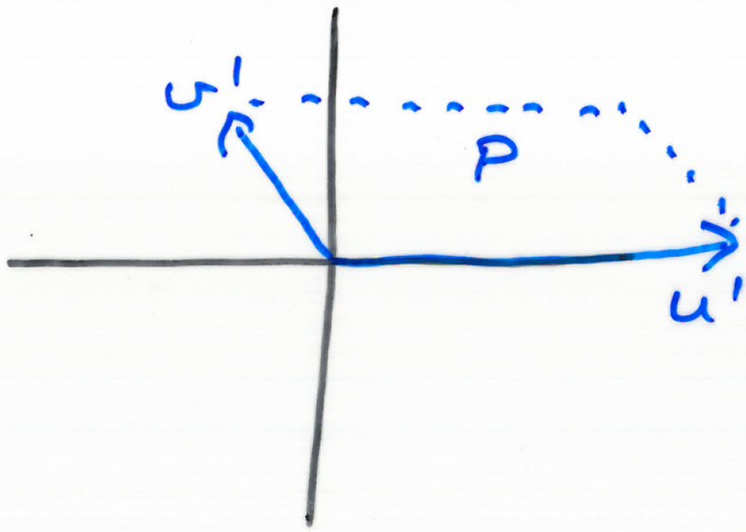
$$|AB| = |A||B|.$$

Easy Proposition : For square matrices A, B we have

$$|AB| = |A||B| .$$

Towards a proof that the areas of parallelograms are captured by determinants.





now

$$u = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} u'$$

$$v = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} v'$$

So

$$\det\begin{pmatrix} | & | \\ u & v \\ | & | \end{pmatrix}$$

$$= \det\left(\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} | & | \\ u' & v' \\ | & | \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \det\begin{pmatrix} | & | \\ u' & v' \\ | & | \end{pmatrix}$$

$$= (\cos^2\theta + \sin^2\theta)\,(\pm \text{ area of } P)$$

from last lecture

$$= \pm \text{ area of } P.$$

Eigenvalues & Eigenvectors

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 matrix of real numbers.

Definition A non-zero vector

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

is an eigenvector for A if there is some real number λ such that

$$A v = \lambda v.$$

We call λ the eigenvalue corresponding to v.

<u>Example</u>

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Consider

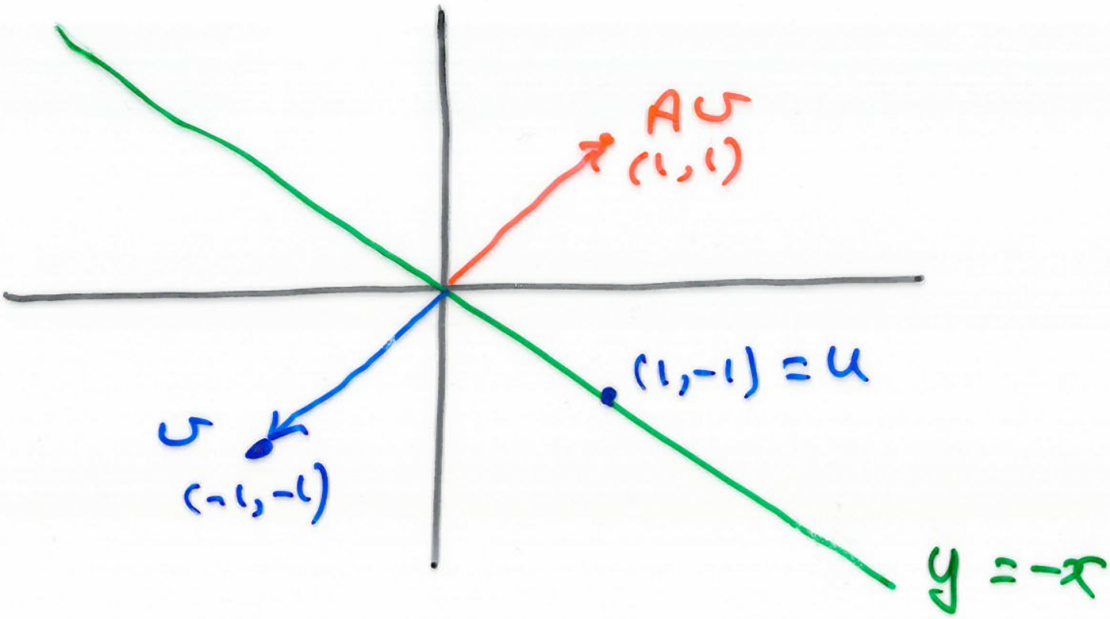
$$v = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Then

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \end{pmatrix} = 3 \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Thus v <u>is</u> an eigenvector for A with eigenvalue $\lambda = 3$.

<u>Example</u> Let A be the matrix of reflection in the line $y = -x$.



$Av = -1 \cdot v$ for $v = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

So v is an eigenvector for A with eigenvalue $\lambda = -1$.

$Au = u$ for $u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. So $u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is an eigenvector for A with eigenvalue $\lambda = 1$.

<u>Example</u> Give me a matrix A that has no eigenvectors.

Let A be the matrix of rotation through $90°$ about the origin, clockwise. This A has no eigenvectors.