$$\left.\begin{aligned} \boxed{2}x + 3y + 2z &= 100 \\ x + y + 4z &= 70 \\ 20x + 10y + 10z &= 500 \end{aligned}\right\} \text{system of linear equations}$$

The system is equivalent to the following system:

$$\begin{bmatrix} R_2 \longmapsto R_2 - \frac{1}{2} R_1 \\ R_3 \longmapsto R_3 - 10 R_1 \end{bmatrix}$$

$$\begin{aligned} 2x + 3y + 2z &= 100 \\ \boxed{-\tfrac{1}{2}}y + 3z &= 20 \\ -10y - 10z &= -500 \end{aligned}$$

This second system is equivalent to:

$$[R_3 \longmapsto R_3 - 40 R_2]$$

$$2x + 3y + 2z = 100$$

$$-\tfrac{1}{2}y + 3z = 20$$

$$-130z = -1300$$

Back substitution :

$z = 10$

$y = 20$

$x = 10$

Notation :

2 is the pivot in the first stage

$-\frac{1}{2}$ " " " " " second "

The above procedure for solving a system of linear equations is called Gaussian Elimination.

Q: Does this procedure always work?

A: It doesn't work if a pivot at some stage is zero.

Topic 3

Determinants, Eigenvalues & Eigenvectors

(Mainly 2x2 matrices)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Definition The adjoint matrix of A is

$$\operatorname{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Observe:

$$A \cdot \mathrm{adj}(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= (ad-bc) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

<u>Definition</u> The <u>determinant</u> of A is the number

$$\det(A) = ad - bc$$

<u>Note:</u>

$$A \cdot \mathrm{adj}(A) = \det(A) \cdot I$$

or

$$A \left(\frac{1}{\det(A)} \mathrm{adj}(A) \right) = I$$

"Thus"

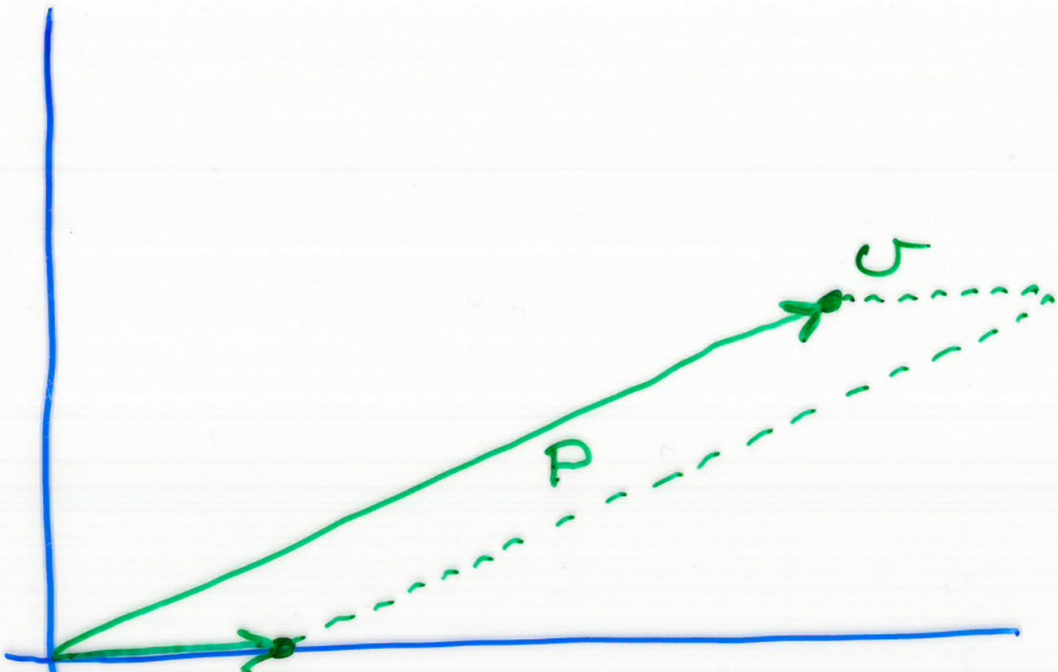
Proposition If $\det(A) \neq 0$ then

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

for any 2×2 matrix.

Consider two "random" vectors

$$u = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$



So u, v determine a parallelogram.

Area of P = base $\times$ $\perp^r$ height

$= 2 \times 3$

$= 6$

Consider

$$A = \begin{pmatrix} 2 & 6 \\ 0 & 3 \end{pmatrix}$$

The vectors u, v can be thought of as the columns of a matrix A.

$$\det(A) = 2.3 - 0.6 = 6.$$

<u>Theorem</u> The determinant of a 2×2 matrix is equal to the area of the parallelogram determined by its two columns.