Punch line from yesterday:

Theorem Let

$S : \mathbb{R}^2 \to \mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$

be linear transformations.

Suppose $S$ is represented by a matrix $A$.

Suppose $T$ is represented by a matrix $B$.

Then, the composite transformation

$$T \circ S : \mathbb{R}^2 \to \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto T\left(S\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

is represented by the matrix

$$BA$$

# Matrix Multiplication

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 3 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

so $AB = I$

so $A^{-1} = B = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$

1) where did h get B ?

2) who cares & why ?

# 1) Gauss-Jordan method for finding the inverse of A

$$(A \mid I) \xrightarrow[\text{operations}]{\text{row}} (I \mid B)$$

Then $B = A^{-1}$.

There are three types of allowable row operation:

(I) $R_i \longmapsto R_i + \lambda R_j \quad (j \neq i)$

(II) $R_i \longleftrightarrow R_j$

(III) $R_i \longleftrightarrow \lambda R_i \quad (\lambda \neq 0)$

To find $A^{-1}$ :

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \mapsto R_2 - 2R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \mapsto R_3 - 3R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -3 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \mapsto R_3 - 2R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right) \xrightarrow{R_3 \mapsto -1 R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right) \xrightarrow[R_1 \mapsto R_1 - 3R_3]{R_2 \mapsto R_2 + R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & -6 & 3 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right) \xrightarrow{R_1 \mapsto R_1 - 2R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -12 & 5 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right)$$

Thus

$$A^{-1} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

(2) who cares

$$\left.\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 5y + 5z &= 2 \\ 3x + 8y + 6z &= 3 \end{aligned}\right\} (*)$$

Can be written as

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$