MA180, MA190, MA185 Algebra

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Topics	Content
• Elementary number theory	Internet communications
• Matrix Theory	Geometry & Internet communications
• Eigenvalues & Eigenvectors	Breeding rabbits

Elementary number theory

$5 + 15 = 20$ in school

$7 + 10 = 5$ on a clock

$7 + 10 \equiv 5 \mod 12$

Today is Monday.
In 73 days time it will be Thursday

$73 + 1 \equiv 4 \mod 7$

More examples

$10 \times 5 \equiv 2 \mod 12$

$$7 + 5 \equiv 3 \mod 9$$

$$7 \times 8 \equiv 2 \mod 9$$

$$2 - 5 \equiv 5 \mod 8$$

What is $\frac{1}{3}$?

$\frac{1}{3}$ is that number with the property

$$\left(\frac{1}{3}\right) \times 3 = 1$$

Alternative notation

$$3^{-1} = \frac{1}{3}$$

We call 3^{-1} the inverse of 3.

Back to clocks

What is $7^{-1} \mod 10$?

$$7^{-1} \equiv 3 \mod 10$$

because $7 \times 3 \equiv 1 \mod 10$.

Application

Any book is identified by an ISBN. On older books this is a string of 10 digits.

0-141-31135-5

James and the giant peach.

The final digit is chosen so that

$$0\times1 + 1\times2 + 4\times3 + 1\times4 + 3\times5$$

$$+ 1\times6 + 1\times7 + 3\times8 + 5\times9$$

$$+ 5\times10$$

$$\equiv 0 \pmod{11}$$