

MA180, MA190, MA185 Algebra

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Topics

- Elementary number theory
- Matrix theory
- Eigenvalues & Eigenvectors

Context

Internet communications

Geometry & internet communications

Breeding rabbits & beauty

Elementary number theory

$$10 + 8 = 18$$

in
School

$$10 + 8 = 6$$

on a
clock

$$10 + 8 \equiv 6 \pmod{12}$$

Today is Wednesday.
in 73 days time it
will be Saturday

$$73 \equiv 3 \pmod{7}$$

More examples

$$10 \times 5 \equiv 2 \pmod{12}$$

$$7 + 5 \equiv 3 \pmod{9}$$

$$7 \times 8 \equiv 2 \pmod{9}$$

$$2 - 5 \equiv 5 \pmod{8}$$

What is $\frac{1}{3}$?

$\frac{1}{3}$ is that number with the property that

$$\left(\frac{1}{3}\right) 3 = 1.$$

Alternative notation

$$3^{-1} = \frac{1}{3}$$

We call 3^{-1} the inverse of 3.

Back to clock

What is $7^{-1} \pmod{10}$?

$$7^{-1} \equiv 3 \pmod{10}$$

because $7 \times 3 \equiv 1 \pmod{10}$.

Application

Any book is identified by an ISBN. On older books this is a string of 10 digits.

0-671-02736~~8~~-0

identifies

"Angels & Demons"

The last digit of the ISBN
is chosen such that

$$0 \times 1 + 6 \times 2 + 7 \times 3 + 1 \times 4 + 0 \times 5$$

$$+ 2 \times 6 + 7 \times 7 + 3 \times 8 +$$

$$+ 6 \times 9 + 0 \times 10$$

$$\equiv 0$$

$$\text{mod } 11$$