Problem

You intercept the ciphertext

OH7F86BB46R3627026BB9

and you know:

1) A 37-letter alphabet was used

$\emptyset, 1, 2, 3, \cdots, 9, A=10, B=11, \cdots, Z=35, -=36$

2) An affine cryptosystem

$$X \longmapsto \alpha X + \beta$$

is used on single letter message units with enciphering key (α, β).

3) Plaintext ends with $\emptyset\emptyset 7$

Decipher the message.

Solution

Enciphering procedure

$\phi \longmapsto 11$

$7 \longmapsto 9$

$11 = \alpha \cdot \phi + \beta \pmod{37}$

$9 = \alpha \cdot 7 + \beta$

Thus

$\boxed{\beta = 11}$

$9 = 7\alpha + \beta \pmod{37}$

$-2 = 7\alpha \pmod{37}$

$35 = 7\alpha \pmod{37}$

"So"

$\boxed{\alpha = 5}$

The enciphering function is

$X \longmapsto 5X + 11 \pmod{37}$

The deciphering function is

$$X \mapsto 5^{-1}(X - 11) \mod 37$$

To calculate $5^{-1} \mod 37$ we first note that

$$\gcd(5, 37) = 1$$

Let's use the Euclidean algorithm to compute $\gcd(5, 37)$.

$$37 = 7.5 + 2$$

$$5 = 2.2 + 1 = \gcd(5, 37)$$

So

$$1 = 5 - 2.2$$

$$= 5 - 2(37 - 7.5)$$

$$= 15.5 - 2.37$$

$$\equiv 15.5 \mod 37$$

So $\boxed{5^{-1} = 15 \mod 37}$

Deciphering function is

$$X \longmapsto 5^{-1}(x - 11) \mod 37$$

$$= 15(x - 11) \mod 37$$

$$= 15x - 15.11$$

$$= 15x + 20$$