Chinese Remainder Theorem

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 6 \pmod{7}$$

Solve for x.

Let

$$x = 2 \cdot 10 \cdot 7 \qquad \text{Mod } 3$$

$$+$$

$$3 \cdot \cancel{6} \cdot 21 \qquad \text{Mod } 5$$

$$+$$

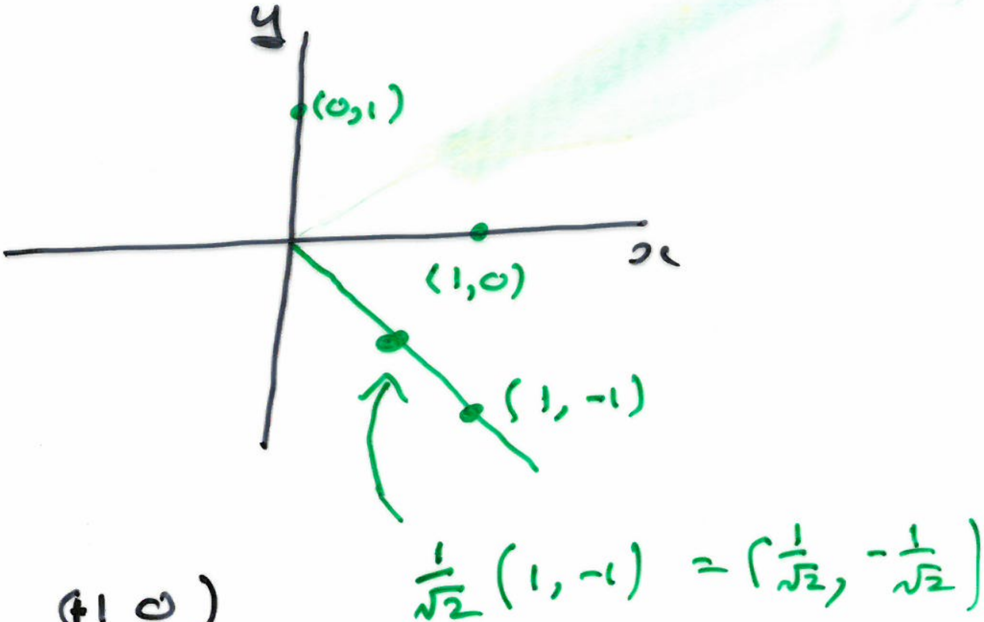
$$6 \cdot 15 \qquad \text{mod } 7$$

$$x = 140 + 63 + 90 = 293$$

2012/13

Q 3

a)



$f(1,0) = (1,0)$

$f(0,1) = (0,-1)$

Matrix of f is

$$F = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$g(1,0) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$g(0,1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

The matrix of g is

$$G = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Matrix for gof is

$$GF = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

Q3 b)

$$\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix} = \underbrace{\begin{pmatrix} 5 & -6 \\ 1 & 0 \end{pmatrix}}_{A}\begin{pmatrix} x_{n-1} \\ x_{n-2} \end{pmatrix}$$

ii) $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 5-\lambda & -6 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(-\lambda) + 6 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$(\lambda - 3)(\lambda - 2) = 0$$

$$D = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$