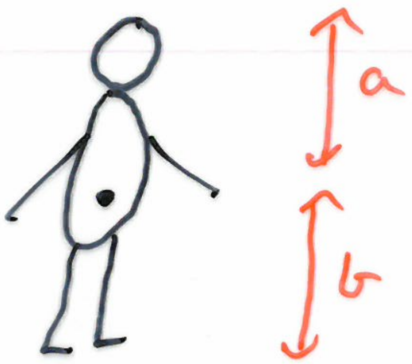
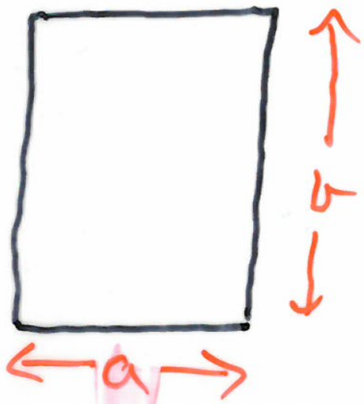
Opinión



A beautiful body is one such that

$$\frac{b}{a} = \phi$$

Opinión



A window is beautiful if $\frac{b}{a} = \phi$.

Biology uses the Golden Ratio.

Yesterday:

F_n = number of pairs of rabbits in a field after n months.

Under certain assumptions:

$$F_0 = 1$$
$$F_1 = 1$$
$$F_2 = 2$$
$$F_3 = 3$$
$$F_4 = 5$$
$$F_5 = 8$$
$$\vdots$$

$$\boxed{F_n = F_{n-1} + F_{n-2}}$$

A calculator experiment suggested

$$\lim_{n \to \infty} \frac{F_n}{F_{n-1}} \quad \text{exists.}$$

Let's suppose

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \phi$$

exists. This would mean that the population increases (roughly) by a factor of ϕ each month.

$$\boxed{\begin{aligned} F_n &= F_{n-1} + F_{n-2} \\ F_{n-1} &= F_{n-1} \end{aligned}}$$

$$\begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n-1} \\ F_{n-2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n-2} \\ F_{n-3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} F_{n-2} \\ F_{n-3} \end{pmatrix}$$

$$\vdots$$

$$\boxed{\begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{n-1} \begin{pmatrix} F_1 \\ F_0 \end{pmatrix}}$$

Given $\lim\limits_{n \to \infty} \dfrac{F_n}{F_{n-1}} = \phi$

we see that for large n we roughly have

$$\frac{F_n}{F_{n-1}} \simeq \phi$$

or

$$\boxed{F_n \simeq \phi \, F_{n-1}}$$

or

$$\phi \begin{pmatrix} F_{n-1} \\ F_{n-2} \end{pmatrix} \simeq \begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = \overbrace{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}}^{A} \begin{pmatrix} F_{n-1} \\ F_{n-2} \end{pmatrix}$$

So ϕ would be (roughly)

an eigenvalue for A.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

To find the eigenvalues for A:

$$\det \begin{pmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = 0$$

$$(1-\lambda)(-\lambda) - 1 = 0$$

$$\lambda^2 - \lambda - 1 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\lambda_1 = \frac{1+\sqrt{5}}{2} \simeq 1.618\ldots\ldots$$

$$\lambda_2 = \frac{1-\sqrt{5}}{2}$$

are the eigenvalues of A.

The number

$$\phi = \frac{1+\sqrt{5}}{2}$$

is called the <u>Golden</u> <u>Ratio</u>

<u>Alternative</u> <u>definition</u>: Two quantities $b > a$ are said to be in the <u>Golden</u> <u>Ratio</u> if $\frac{a+b}{b} = \frac{b}{a}$.

Note: if

$$\frac{a+b}{b} = \frac{b}{a} = \lambda$$

then $b = a\lambda$

and

$$\frac{a+a\lambda}{a\lambda} = \lambda$$

so

$$\frac{1+\lambda}{\lambda} = \lambda$$

and

$$1+\lambda = \lambda^2$$

or

$$\lambda^2 - \lambda - 1 = 0 .$$