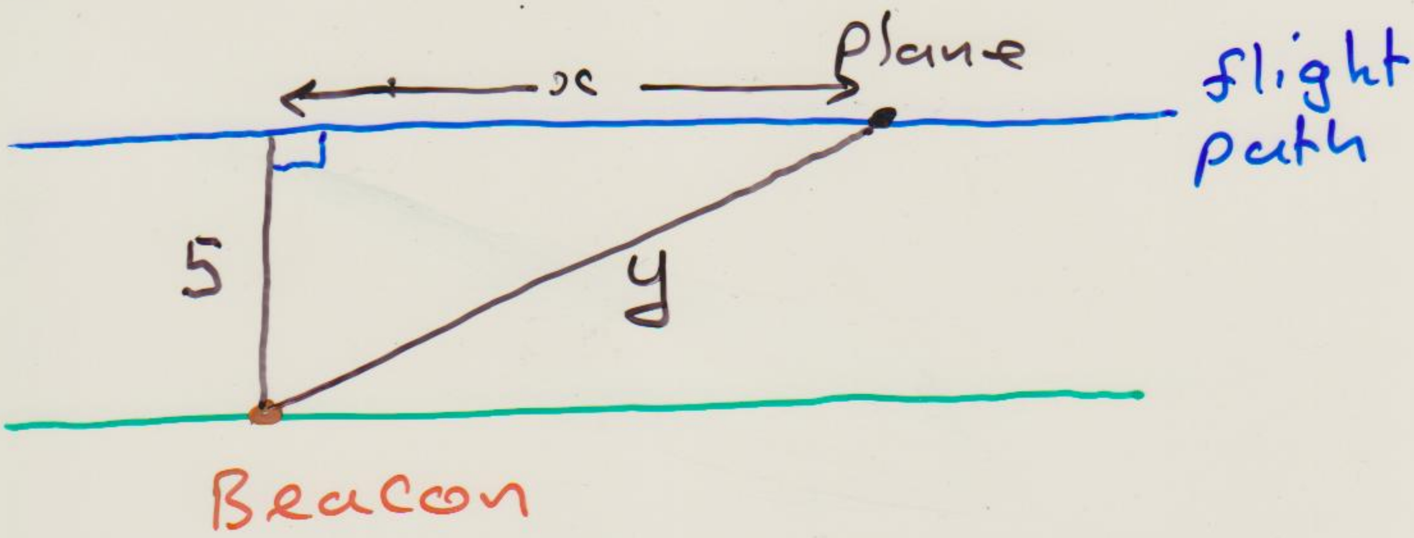
The Real Stuff —
Applications

The derivative can be thought of as a rate of change.

Problem An aircraft is flying horizontally at a speed of 600 km/h. How fast is the distance between the aircraft and a radio beacon increasing 1 minute after the plane passes 5 km directly above the beacon?

Sol[n]



want to find

$$\boxed{\frac{dy}{dt} \text{ when } t = 1.}$$

when $t = 1$, $x = 10$.

$$x^2 + 5^2 = y^2 \qquad (*)$$

and both x and y are functions of t.

chain rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Differentiate both sides of (*) with respect to t.

$$2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt} \quad (**)$$

$\frac{dx}{dt} = 10$

so, when $t = 1$ the eqn (**) becomes

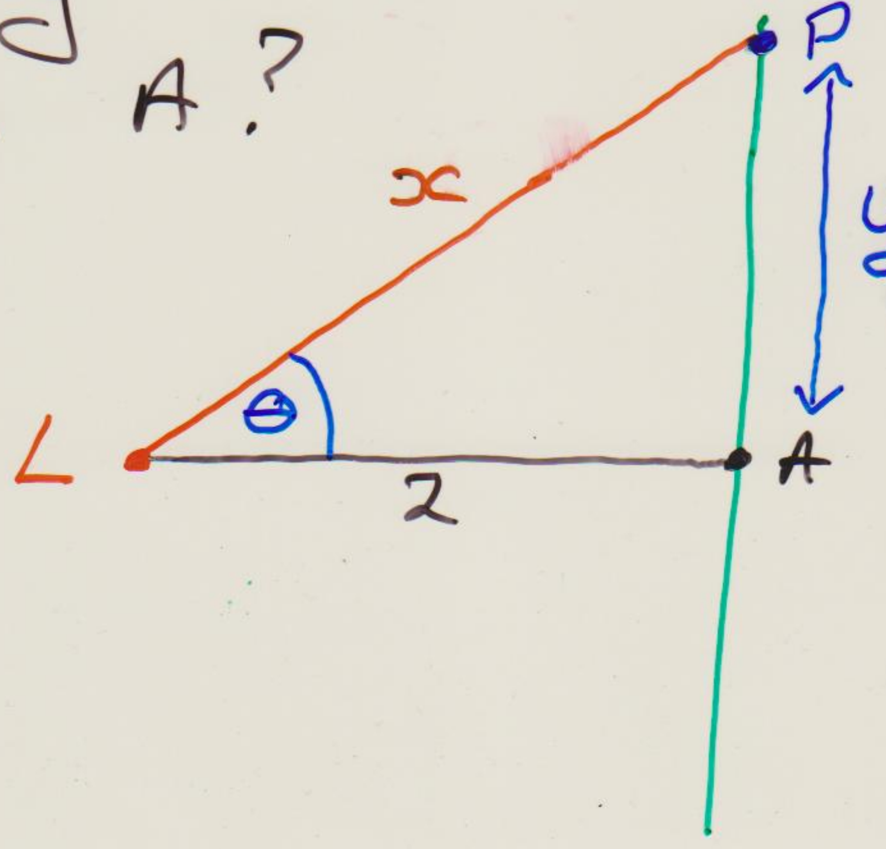
$$2 \cdot 10 \cdot 10 = 2 \cdot \left(\sqrt{5^2 + 10^2}\right) \frac{dy}{dt}$$

$$\frac{\cancel{2} \cdot 100}{\cancel{2} \cdot \sqrt{125}} = \frac{dy}{dt}$$

$$\frac{100}{\sqrt{5 \cdot 25}} = \frac{dy}{dt}$$

$$\frac{20}{\sqrt{5}} = \frac{dy}{dt} \quad \text{km/min}$$

Problem A lighthouse L is located on a small island 2 km from the nearest point A on a long straight shoreline. The lighthouse light rotates at 3 revs per minute. How fast is the illuminated spot P on the shoreline moving when it is 4 km from A?



$\boxed{\text{Need to find } \dfrac{dy}{dt} \text{ when } y = 4.}$

Given

$$\frac{d\theta}{dt} = 6\pi \quad \text{radians/min}$$

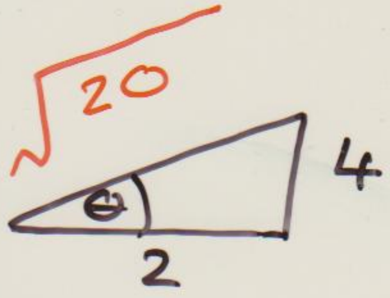
Both θ and y are functions of t.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{2}$$

Differentiate both sides w.r.t. t.

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt} \quad (*)$$

when $y = 4$



$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{\sqrt{20}}{2}$$

Hence, from (*), when $y=4$ we have

$$\frac{20}{4} \cdot 6\pi = \frac{1}{2}\frac{dy}{dt}$$

$$60\pi = \frac{dy}{dt} \quad \text{km/min}$$