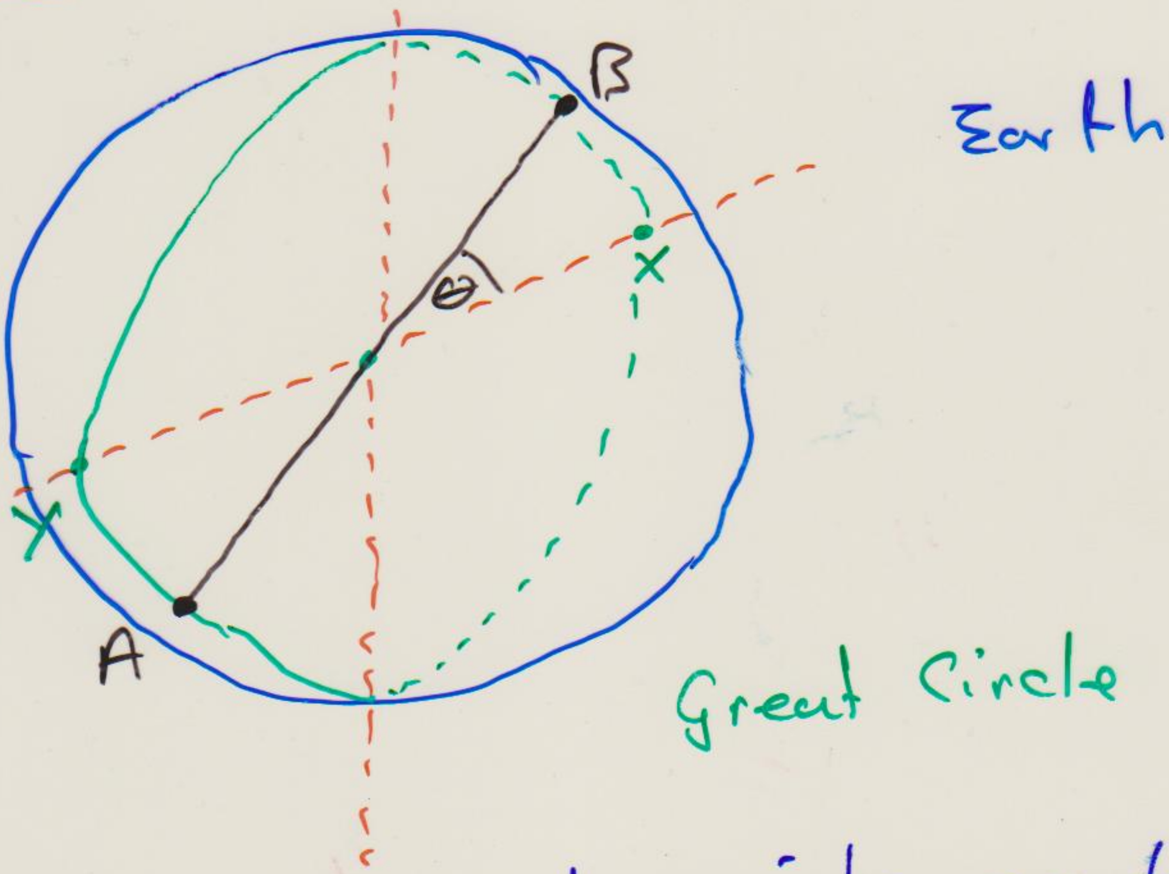
Exams

- Sem I exam (2 hrs)
- Sem II exam (2 hrs)
- CA

Application of IVT



Take any great circle on the earth.

FACT: There exist two opposite points on the great circle with equal air pressure.

Explanation of fact

Consider

$f(\theta) =$ air pressure at A

$\quad -$ air pressure at B

Note: $f(\theta)$ is a continuous function of θ.

I want to prove that for some angle θ the pressure at A = pressure at B.

i.e. want to prove that $f(\theta) = 0$ for some angle $\theta \in [0, \pi]$.

If $f(0) = 0$, or if $f(\pi) = 0$, then we are done!

Suppose then that
$f(0) \neq 0$ and $f(\pi) \neq 0$.

Note:
$f(0) f(\pi) < 0$

($f(0)$ = air pressure at Y
- air pressure at X.

$f(\pi)$ = air pressure at X
- air pressure at Y.

$f(\pi) = -f(0)$, so $f(\pi) f(0) < 0$.)

Intermediate value Theorem
says: There exists at
least one angle $\theta \in [0, \pi]$
such that

$f(\theta) = 0$.

QED

Limits at infinity

Example Evaluate

$$l = \lim_{x \to \infty} \frac{1}{\sqrt{x^2 + 2x} - x}$$

Sol^n

$$l = \lim_{x \to \infty} \left(\frac{1}{\sqrt{x^2+2x} - x} \right) \frac{(\sqrt{x^2+2x} + x)}{(\sqrt{x^2+2x} + x)}$$

$$l = \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{x^2+2x - x^2}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{2x}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2+2x}}{\sqrt{4x^2}} + \frac{1}{2}$$

$$= \lim_{x \to \infty} \sqrt{\frac{x^2+2x}{4x^2}} + \frac{1}{2}$$

$$\lim_{x \to \infty} \sqrt{\frac{1}{4} + \frac{1}{2x}} + \frac{1}{2}$$

$$l = \sqrt{\frac{1}{4}} + \frac{1}{2}$$

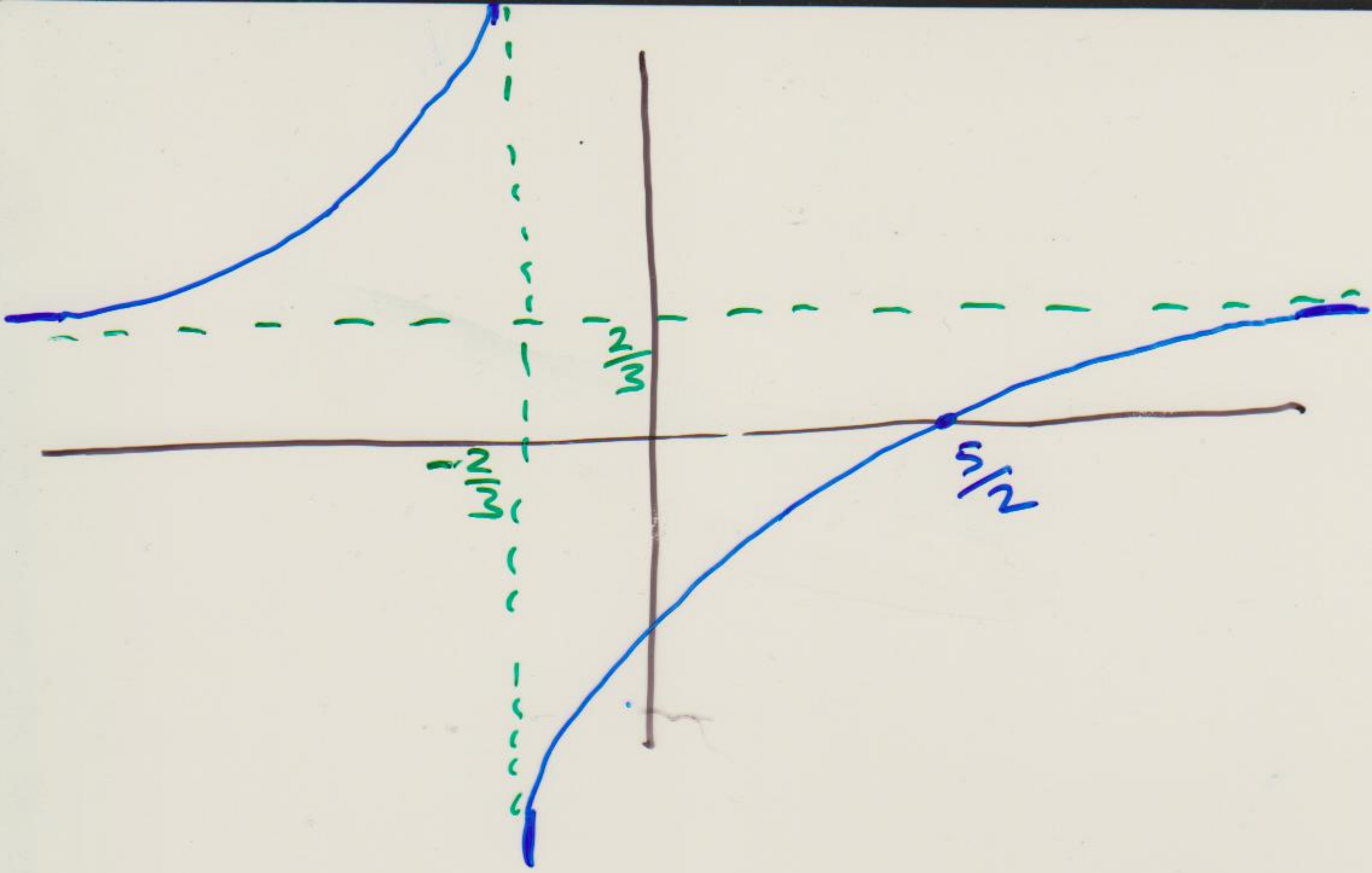
$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

<u>Example</u> what are the horizontal & vertical asymptotes of

$$y = \frac{2x - 5}{3x + 2} \ ?$$

Then sketch the graph of y.



$$\lim_{x \to \infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

$$\lim_{x \to -\infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

we get a vertical asymptote when $3x+2=0$, $x = -\frac{2}{3}$

$y=0$ when $2x-5=0$ or $x = \frac{5}{2}$

So: Limits at infinity correspond to horizontal asymptotes.