Consider $g(x) = \dfrac{x^6 - 1}{x - 1}$

Domain $= \mathbb{R} - \{1\}$

$$g(0.9) = \frac{(0.9)^6 - 1}{0.9 - 1} = 4.68$$

$$g(1.01) = \frac{(1.01)^6 - 1}{1.01 - 1} = 6.15$$

$$g(0.99) = \frac{(0.99)^6 - 1}{0.99 - 1} = 5.85$$

$$g(0.999) = \frac{(0.999)^6 - 1}{0.999 - 1} = 5.99$$

We write

$$\lim_{x \to 1} g(x) = 6$$

to mean that for all numbers x sufficiently close to 1, but distinct from 1, the value of $g(x)$ is close to 6.

Example

Evaluate

$$l = \lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x}$$

Sol^n

For $x \neq 0$

$$\frac{\sqrt{4+x} - 2}{x} \cdot \frac{(\sqrt{4+x} + 2)}{(\sqrt{4+x} + 2)}$$

$$= \frac{\cancel{4} + \cancel{x} - \cancel{4}}{\cancel{x}(\sqrt{4+x} + 2)}$$

$$= \frac{1}{\sqrt{4+x} + 2}$$

Thus

$$l = \lim_{x \to 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{4}$$

Example

Evaluate

$$\ell = \lim_{x \to 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right)$$

Sol^n for $x \neq 2$

$$\frac{1}{x-2} - \frac{4}{x^2-4}$$

$$= \frac{x+2-4}{(x-2)(x+2)}$$

$$= \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$$

$$\ell = \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{4}$$

Example

Evaluate

$$l = \lim_{x \to 0} \frac{x}{|x-1| - |x+1|}$$

Example For $x \neq 0$ and x is close to 0

$$\frac{x}{|x-1| - |x+1|}$$

$$= \frac{x}{-(x-1) - (x+1)}$$

$$= \frac{x}{-x+1-x-1} = \frac{x}{-2x} = -\frac{1}{2}$$

$$l = \lim_{x \to 0} \left(-\frac{1}{2}\right) = -\frac{1}{2}.$$

Correct definition of a limit.

We write

$$\lim_{x \to 1} g(x) = 6$$

to mean that:

for any number $\varepsilon > 0$ there exists a $\delta > 0$ such that

$0 < |x - 1| < \delta$

implies

$|g(x) - 6| < \varepsilon$.