Definitión (Joe Biden)

An inflection point. That's the point at which, like, you're driving your car, where the steering wheel is dead straight, and once you make a move, even to a degree, you commit that automobile hurdling in a direction you can't immediately change.

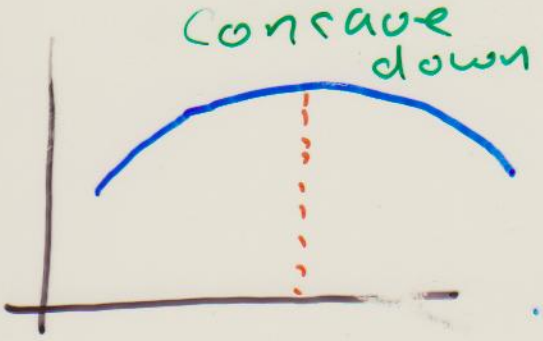
Richard Nixon, 1972, uses a third derivative.

"The rate of increase of inflation is decreasing..."

Recall

$y = f(x)$

local maxima look like



or

y' does not exist here

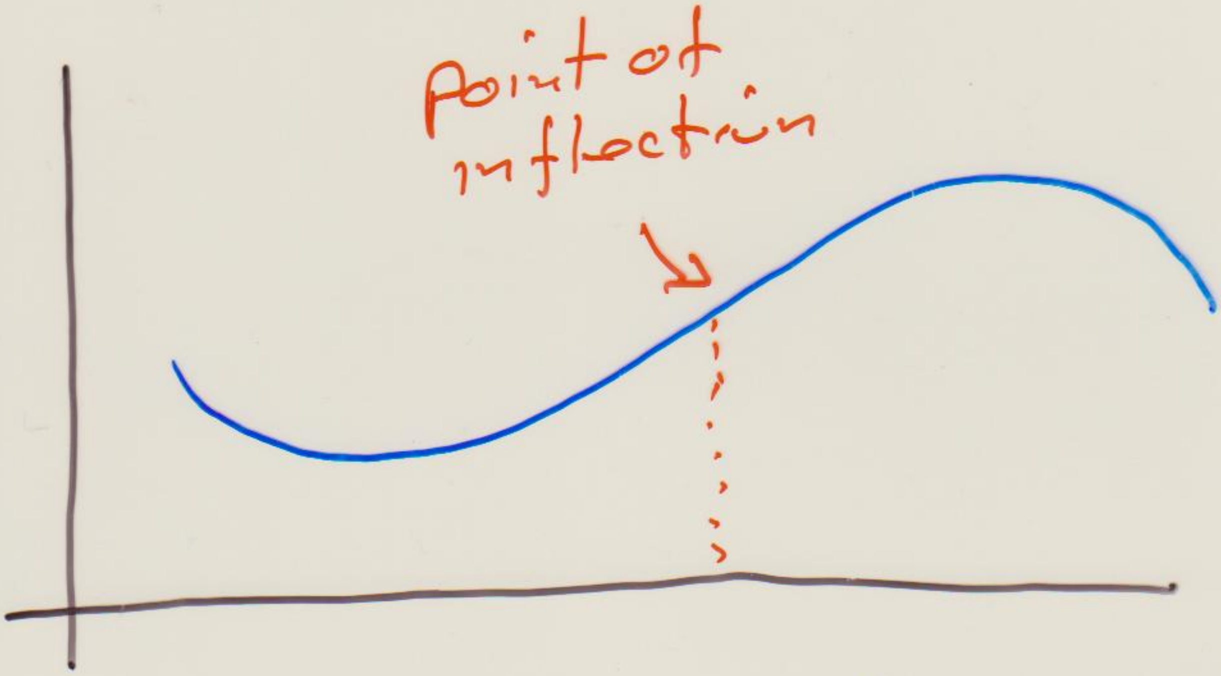
y' $+ + + + 0 - - - - -$

y'' $- - - - - - - - -$

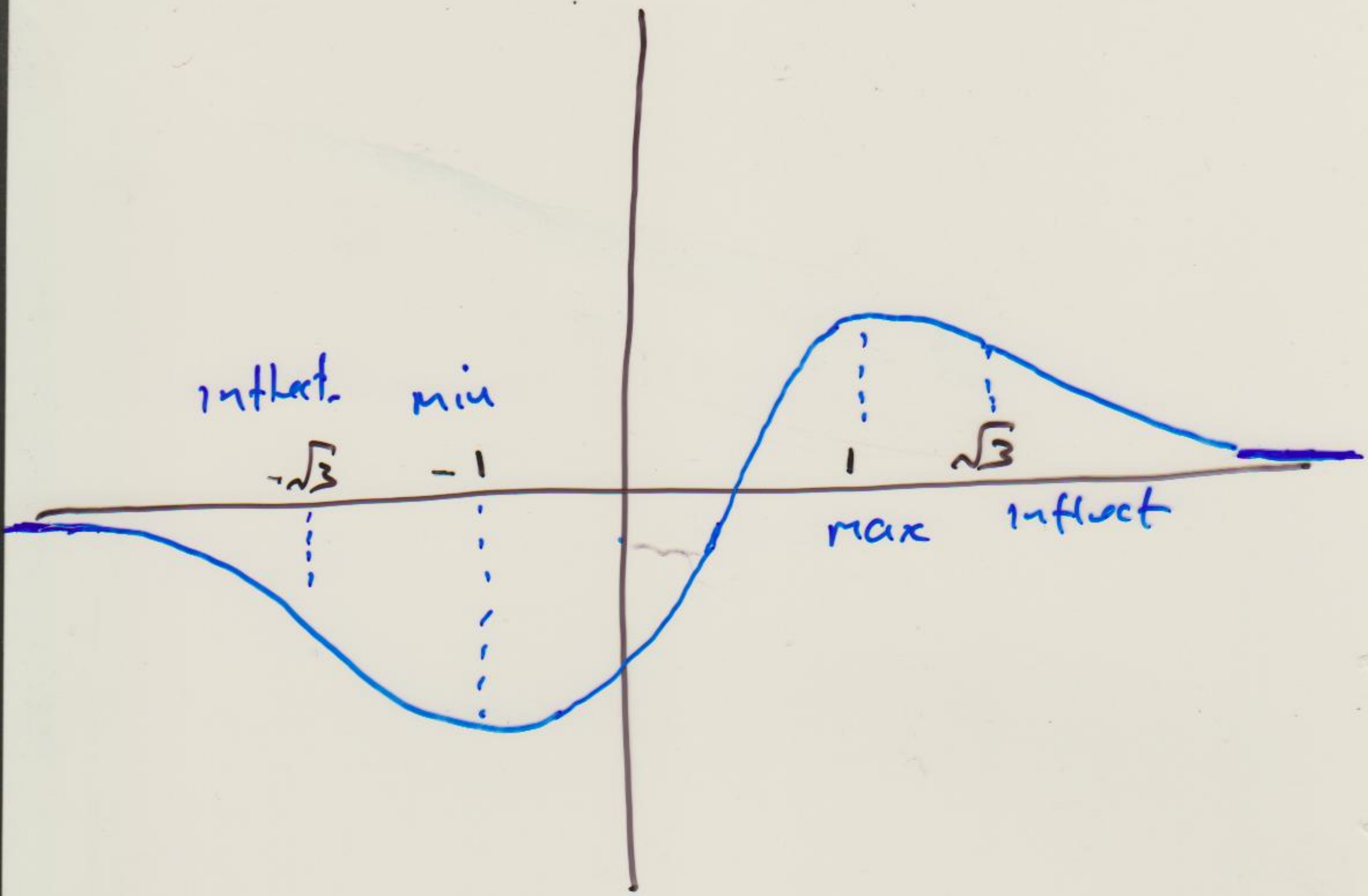
similar pictures for local minima.

if y'' is $\begin{cases} \text{positive} \\ \text{negative} \end{cases}$ on an interval then the function is said to be $\begin{cases} \text{concave up} \\ \text{concave down} \end{cases}$

<u>Defn</u> A point x at which $y = f(x)$ changes from concave up to concave down, or from concave down to concave up, is called a point of <u>inflection</u>



<u>Problem</u> Sketch the graph of $y = x e^{-\frac{x^2}{2}}$, indicating any local maxima, minima and points of inflection.



Domain of $y = f(x)$ is $\mathbb{R}$.

$$\lim_{x \to \infty} x e^{-\frac{x^2}{2}} = \lim_{x \to \infty} \frac{x}{e^{\frac{x^2}{2}}} = 0$$

$$\lim_{x \to -\infty} x e^{-\frac{x^2}{2}} = \lim_{x \to -\infty} \frac{x}{e^{\frac{x^2}{2}}} = 0$$

so the line $y = 0$ is a horizontal asymptote.

$$y = x e^{-\frac{x^2}{2}}$$

$$y' = x e^{-\frac{x^2}{2}}(-x) + e^{-\frac{x^2}{2}}$$

$$\boxed{y' = (1 - x^2) e^{-\frac{x^2}{2}}}$$

$$y'' = (1-x^2) e^{-\frac{x^2}{2}}(-x) + e^{-\frac{x^2}{2}}(-2x)$$

$$y'' = (-x + x^3 - 2x) e^{-\frac{x^2}{2}}$$

$$\boxed{y'' = (x^2 - 3)\, x\, e^{-\frac{x^2}{2}}}$$

local min

pt. inflection

local max

x		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
y'	$- - -$	$-$	$- -$	0	$+ +$	$+$	$+ +$	0	$- -$	$-$	$- - -$
y''	$- - -$	0	$+ +$	$+$	$+ +$	0	$- -$	$-$	$- -$	0	$+ + +$
y	decreasing				increasing				decreasing		
	concave down		concave up				concave down				concave up