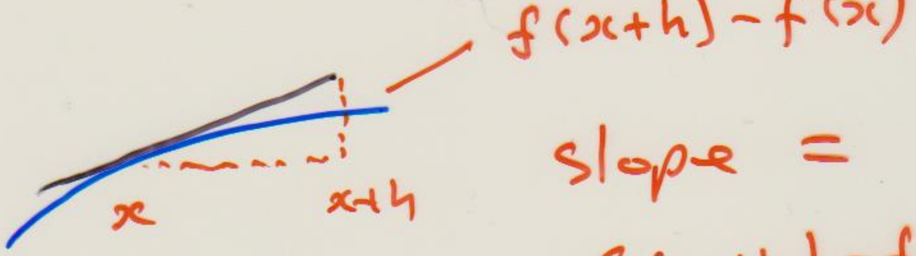
Applications II

Applications where derivatives are used to measure the slope of a tangent to a curve.



slope =

$$\frac{f(x+h)-f(x)}{\cancel{x}+h-\cancel{x}}$$

At points where a continuous function $f(x)$ is a local minimum or local maximum

we have that the derivative

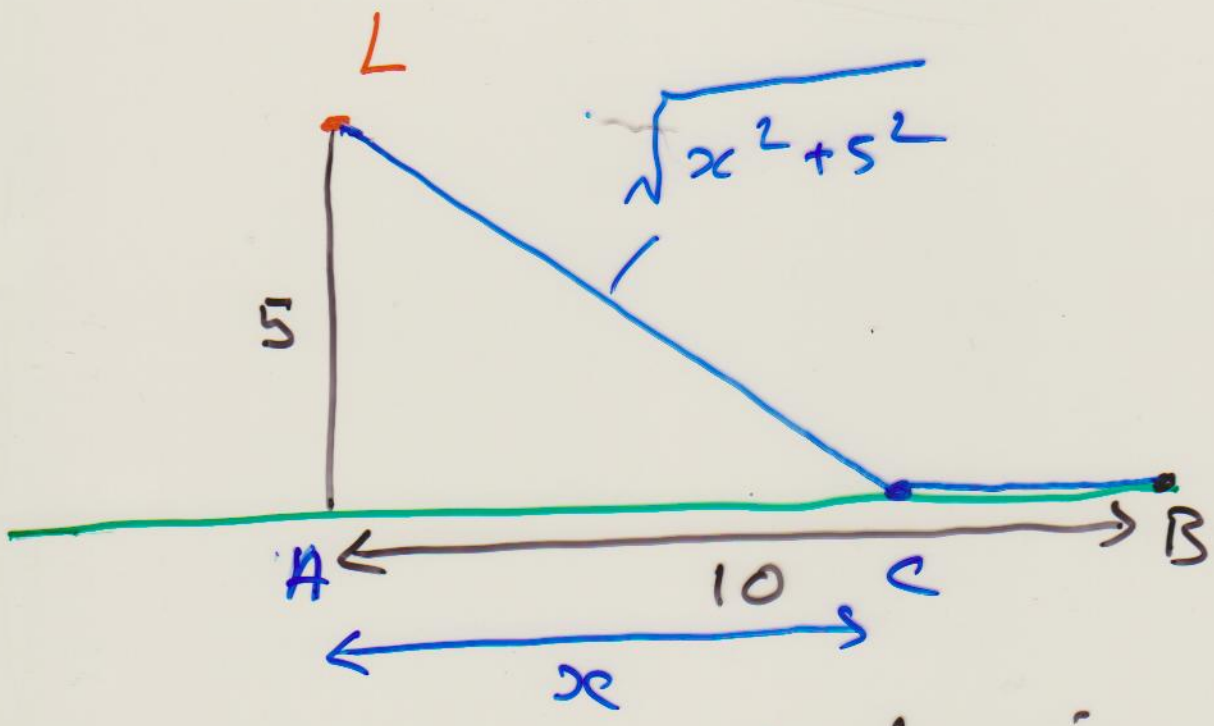
$f'(x) = 0$ or else

$f'(x)$ does not exist.

Problem A lighthouse L is located on a small island 5 km north of a point A on a straight east-west coastline. A cable is to be laid from L to a point B on the coastline 10 km east of A. Laying the cable under water costs €5000 per kilometer. Laying it on land costs €3000 per kilometer.

<u>Question</u> what is the cheapest cost of laying the cable?

<u>Sol^n</u>



Let $f(x)$ = cost of laying the blue cable

$$f(x) = 3000(10-x) + 5000\sqrt{x^2+5^2}$$

$$f(x) = 3000(10-x) + 5000\left(x^2+5^2\right)^{\frac{1}{2}}$$

$$f'(x) = -3000 + \frac{5000}{\cancel{2}\sqrt{x^2+5^2}} \cancel{2}x$$

$$f'(x) = 1000\left(-3 + \frac{5x}{\sqrt{x^2+5^2}}\right)$$

$f'(x) = 0$ when

$$3 = \frac{5x}{\sqrt{x^2+5^2}}$$

or

$$9 = \frac{25x^2}{x^2+5^2}$$

or

$$9x^2 + 9.25 = 25x^2$$

or $\quad 9.25 = 16.x^2$

$$x^2 = \frac{9.25}{16}$$

$$x = \frac{3.5}{4} = \frac{15}{4} \text{ km}$$

Common sense says that $f(x)$ must have a minimum. So the min. occurs at $x = \frac{15}{4}$.

Minimum cost is

$f\left(\frac{15}{4}\right) =$

$3000\left(10 - \frac{15}{4}\right) + 5000\sqrt{\frac{15^2}{4^2} + 5^2}$

euro.