- 1st Homework Deadline: TOMORROW
- student feedback survey

Matrix Arithmetic

School: $+, -, \times, \div$ for integers $\mathbb{Z}$

MA183 so far : $+, -, \times, (\,)^{-1}$ for integers mod m $\mathbb{Z}_m$

Message: $(\,)^{-1}$ is more subtle for $\mathbb{Z}_m$ than for $\mathbb{Z}$. But $+, -, \times$ are essentially the same for $\mathbb{Z}$ and $\mathbb{Z}_m$.

In matrix arithmetic we'll see that $\times$ is also more subtle. In particular, $AB \neq BA$ in general.

A 2x2 matrix is an array

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

2 rows
2 columns

of numbers a, b, c, d.

A 1×2 matrix is an array

$$\begin{pmatrix} a & b \end{pmatrix}$$

1 row
2 columns

of numbers.

A 2×1 matrix is an array

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

2 rows
1 column

Matrix addition

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a+w & b+x \\ c+y & d+z \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a+x \\ b+y \end{pmatrix}$$

Zero matrix

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \end{pmatrix} + \begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} a & b \end{pmatrix}$$

Negative of a matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

so set

$$-\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$$

<u>Note</u> for two matrices A, B of the same dimension we have

$$A + B = B + A .$$

A matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{array}{l} R_1 \\ R_2 \end{array}$$

can be thought of as two rows

$R_1 = (a, b)$, $\quad R_2 = (c, d)$

Multiplication of an $m \times n$ matrix and an $n \times p$ matrix

$$\begin{array}{c} R_1 \\ R_2 \end{array}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\ \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{pmatrix} w & x \\ y & z \end{pmatrix} \end{array} = \begin{pmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \end{pmatrix}$$

$$A \qquad\qquad B \qquad\qquad AB$$

e.g.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ -6 & 11 \end{pmatrix}$$

$$2 \times \cancel{2} \qquad \cancel{2} \times 2 \qquad 2 \times 2$$

In general

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} & \\ & \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

$$m \times \cancel{n} \qquad \cancel{n} \times p \qquad m \times p$$

A matrix

$$X = \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{pmatrix} w & x \\ y & z \end{pmatrix} \end{array}$$

can also be regarded as two columns.

$$c_1 = \begin{pmatrix} w \\ y \end{pmatrix} \qquad c_2 = \begin{pmatrix} x \\ z \end{pmatrix}$$

<u>Multiplication of a row and column</u>

$$\underset{1\times 2}{(a,\ b)} \underset{2\times 1}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{1\times 1}{ax + by}$$

e.g. $(1, 2)\begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3 + 8 = 11.$

Identity matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}$$

$\uparrow$

identity matrix I

Inverse of a square matrix

Let A be a $n \times n$ matrix.

The inverse of A is a $n \times n$ matrix A^{-1} such that

$$A^{-1} A = I$$

(and $AA^{-1} = I$)

Consider

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \mod 5$$

$$\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \mod 5$$

$$\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

So

$$A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$