Operation III

e.g. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix} \xrightarrow{R_2 \to 3R_2} \begin{pmatrix} 1 & 2 & 3 \\ 6 & 15 & 15 \\ 3 & 8 & 6 \end{pmatrix}$

Also

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 6 & 15 & 15 \\ 3 & 8 & 6 \end{pmatrix}$$

In general

if $A \xrightarrow[R_i \to \lambda R_i]{} B$

then

$$E_{ii}^{\lambda} A = B$$

where the matrix E_{ii}^{λ} has:

: λ in i^{th} row and i^{th} column

: 1 on all other diagonal entries

: 0 elsewhere.

Let's look again at the Gauss-Jordan method for inverting a matrix A.

If

$$(A \,\vdots\, I) \xrightarrow[\text{ops}]{\text{row}} (I \,\vdots\, B)$$

then there are matrices $E_1, E_2, \dots, E_k$ such that

$$(E_k \cdots E_2 E_1) A = I .$$

So

$$(E_k \cdots E_2 E_1) A A^{-1} = I A^{-1}$$

and

$$(E_k \cdots E_2 E_1) I = A^{-1} \quad (*) .$$

Note: $(*)$ says that $B = A^{-1}$.

This (kind of) proves that the Gauss Jordan method works.

Example

A factory requires energy, steels and labour to manufacture three machines A, B, C.

Resource	A	B	C	weekly available
Energy	2 MWh	3 MWh	2 MWh	100 MWh
steel	1 tonne	1 tonne	4 tonnes	70 tonnes
labour	20 hrs	10 hrs	10 hrs	500 hrs

What production figures ensure all resources are used?

Soln Let's suppose we manufacture

x units of machine A

y " B

z " C.

If all resources are to be used then

$$
\left.\begin{aligned}
R_1:&\ \boxed{2}x + 3y + 2z = 100\\
R_2:&\ x + y + 4z = 70\\
R_3:&\ 20x + 10y + 10z = 500
\end{aligned}\right\}\text{ system of linear equations}
$$

This is equiv. to the system

$$
\begin{bmatrix}
R_2 \mapsto R_2 - \frac{1}{2}R_1\\
R_3 \mapsto R_3 - 10R_1
\end{bmatrix}
$$

$$
\begin{aligned}
2x + 3y + 2z &= 100\\
\boxed{-\tfrac{1}{2}}y + 3z &= 20\\
-20y - 10z &= -500
\end{aligned}
$$

This is equivalent to the system

$$[R_3 \mapsto R_3 - 40R_2]$$

$$2x + 3y + 2z = 100$$

$$-\tfrac{1}{2}y + 3z = 20$$

$$-130z = -1300$$

Back substitution:

$z = 10$

$y = 20$

$x = 10$.

Notation:

2 is the pivot in the first stage

$-\frac{1}{2}$ " " " " second "

The above procedure is called Gaussian elimination, and can be applied to n equations in n unknowns.

Question: when could this general procedure fail?

Answer: if one of the pivots is zero.