First homework deadline was 11/~~10/2011~~

now 17/10/2011

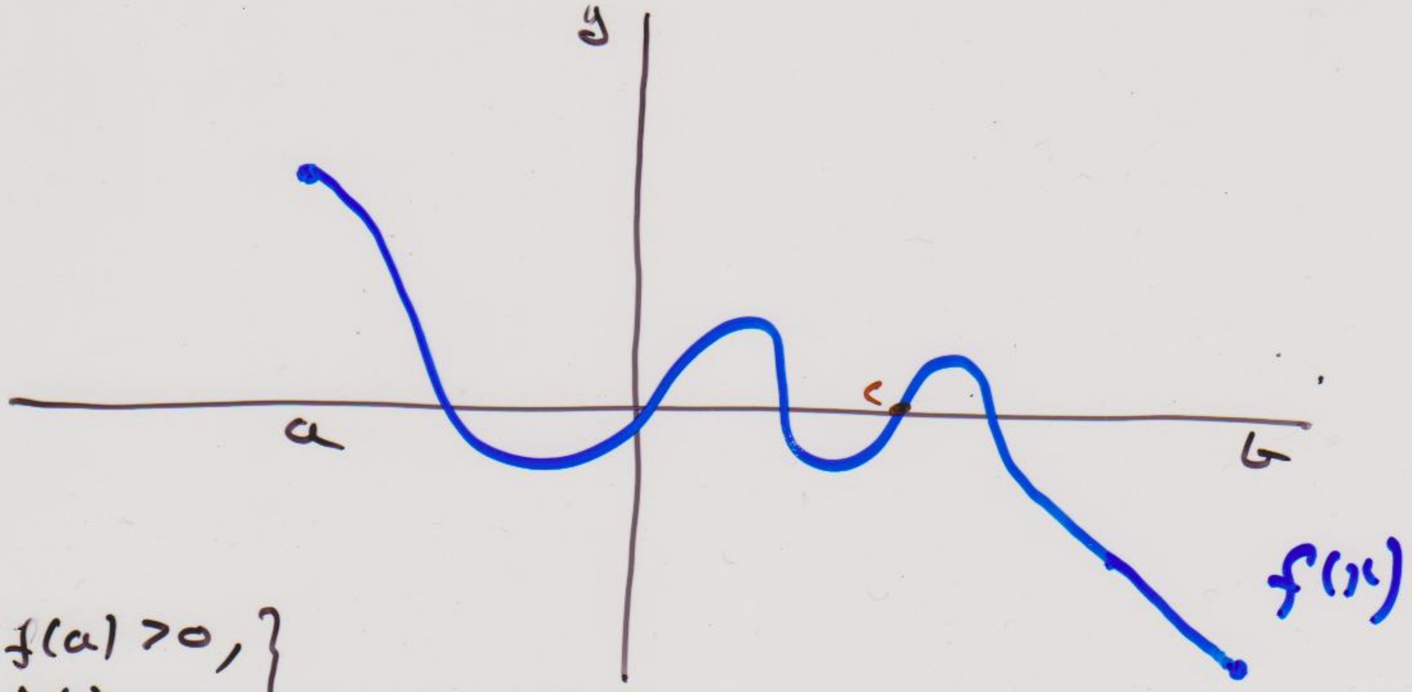
Intermediate Value Theorem

Suppose a function

$$y = f(x)$$

is continuous at all points x in a range $a \le x \le b$. Suppose also that $f(a) f(b) \le 0$. Then there is at least one number c, $a \le c \le b$, such that

$$f(c) = 0.$$



$$\left.\begin{array}{l} f(a) > 0, \\ f(b) < 0 \end{array}\right\} \Rightarrow f(a)f(b) < 0$$

Example Show that

$$x^3 - x - 1 = 0$$

has a solution in the range

$1 \leq x \leq 2$.

Sol^n

Consider

$$f(x) = x^3 - x - 1 .$$

"clearly" $f(x)$ is continuous.

$f(1) f(2) < 0$.

By the IVT there must be some number $1 < c < 2$ such that $f(c) = 0$.

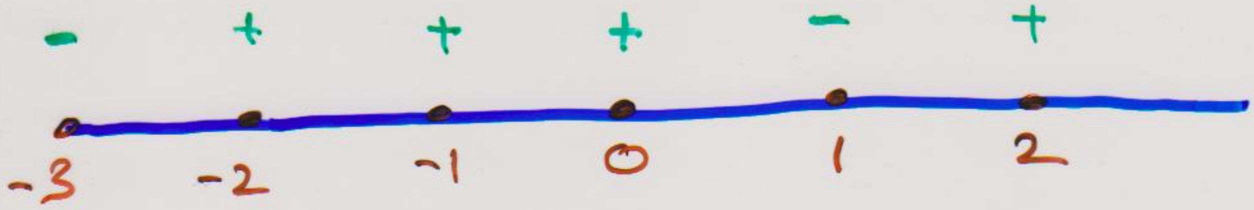
Example

Show that $x^3 - 4x + 1 = 0$ has three (real) solutions.

Sol^n

Consider $f(x) = x^3 - 4x + 1$

Note: $f(x)$ is continuous.



So the IVT say:

$f(0) > 0$

$x^3 - 4x + 1$ has a solution in the interval $1 \leq x \leq 2$.

$f(1) < 0$

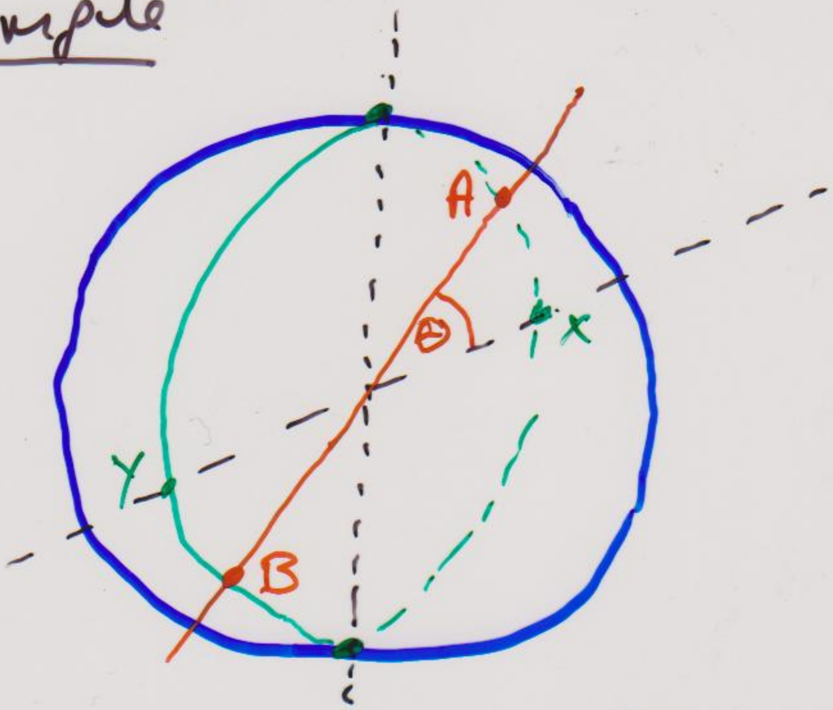
$f(2) > 0$

$f(-2)$

It also has a solution in the interval $0 \leq x \leq 1$.

And also a solution in interval $-3 \leq x \leq -2$.

Example



Take any great circle on the earth.

Fact: there exist two opposite points on the great circle with equal air pressure.

Explanation:
Consider

$$f(\theta) = \text{air pressure at } A - \text{air pressure at } B.$$

Now $f(\theta)$ is a continuous function.

Lets consider $f(\theta)$ in the interval $0 \le \theta \le \pi$.

We want to show that there is some $0 \le c \le \pi$ such that $f(c) = 0$ (for then pressure at A = pressure at B).

If $f(0) = 0$, or $f(\pi) = 0$ then we would be done.

So let's suppose $f(0) \ne 0$, $f(\pi) \ne 0$.

Note:

$$f(0) f(\pi) < 0.$$

(Since $f(0)$ = pressure at X − press at Y
$f(\pi)$ = pressure at Y − press at X)

So IVT says $f(c) = 0$ for some angle c in range $0 \le c \le \theta$.