Don't forget the first homework

Deadline 11th oct (?).

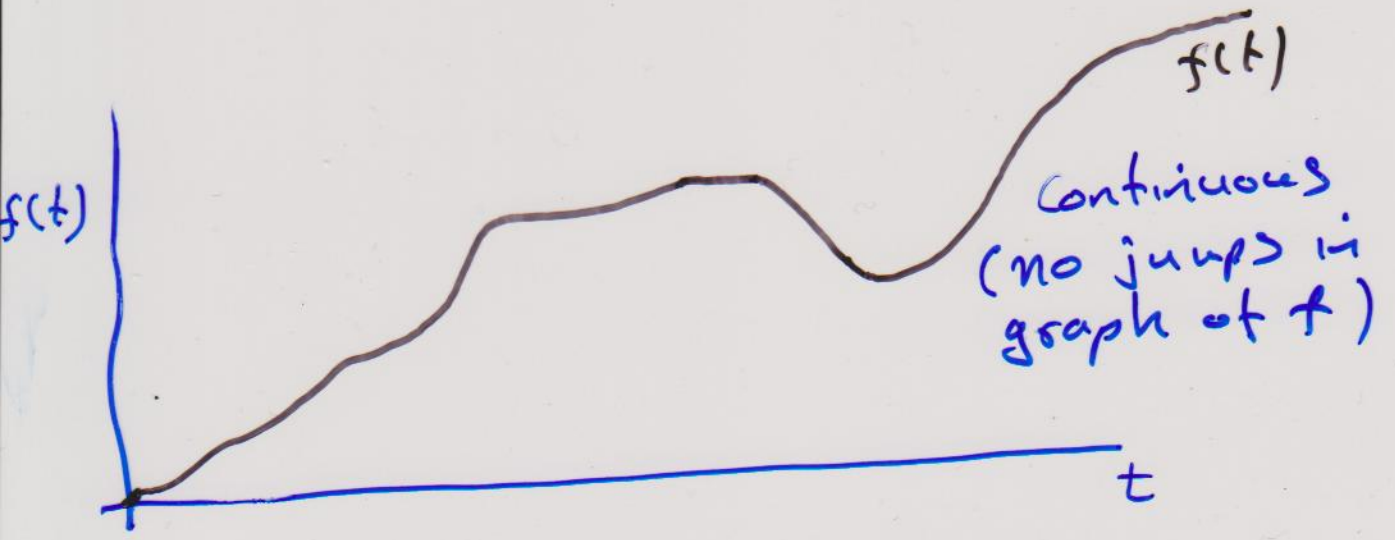
Continuous Functions

(Page 525 in book)

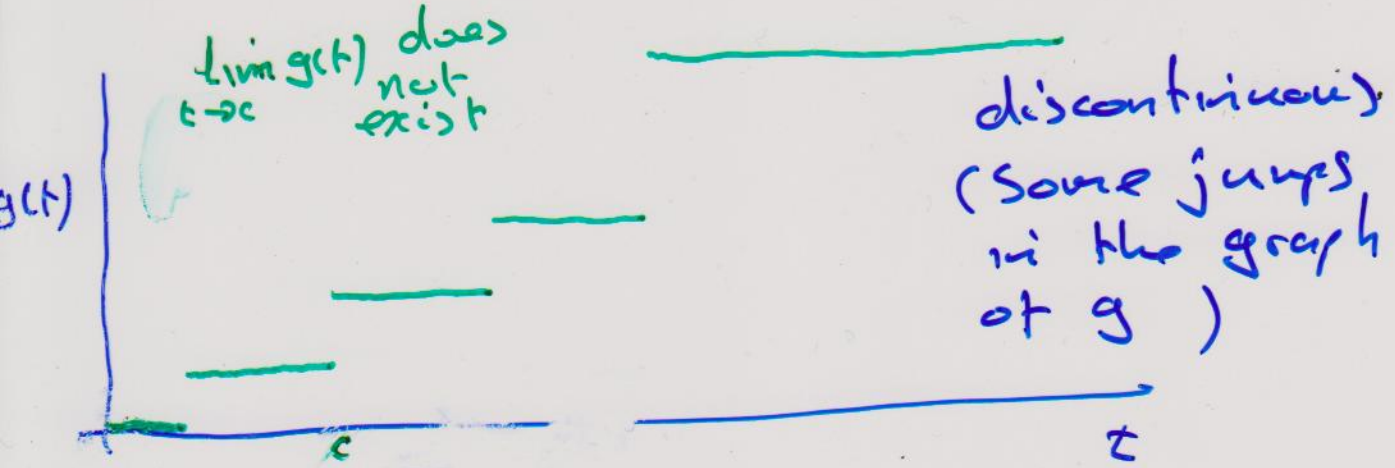
h drive to Dublin airport.

$f(t)$ = my distance from Galway at time t

$g(t)$ = price of my parking ticket at the airport at time t



Continuous (no jumps in graph of f)



discontinuous (some jumps in the graph of g)

Idea A function $f(x)$ is said to be continuous if a small change in the value of x can only result in a small change in the value of $f(x)$.

More precise definition

Suppose $f(x)$ is defined for all values of x near $x = c$ (including $x = c$).

We say that $f(x)$ is continuous at $x = c$ if

$$\lim_{x \to c} f(x) = f(c)$$

Example Consider

$$f(x) = \begin{cases} x & x \geq 0 \\ |x|+1 & x < 0 \end{cases}$$

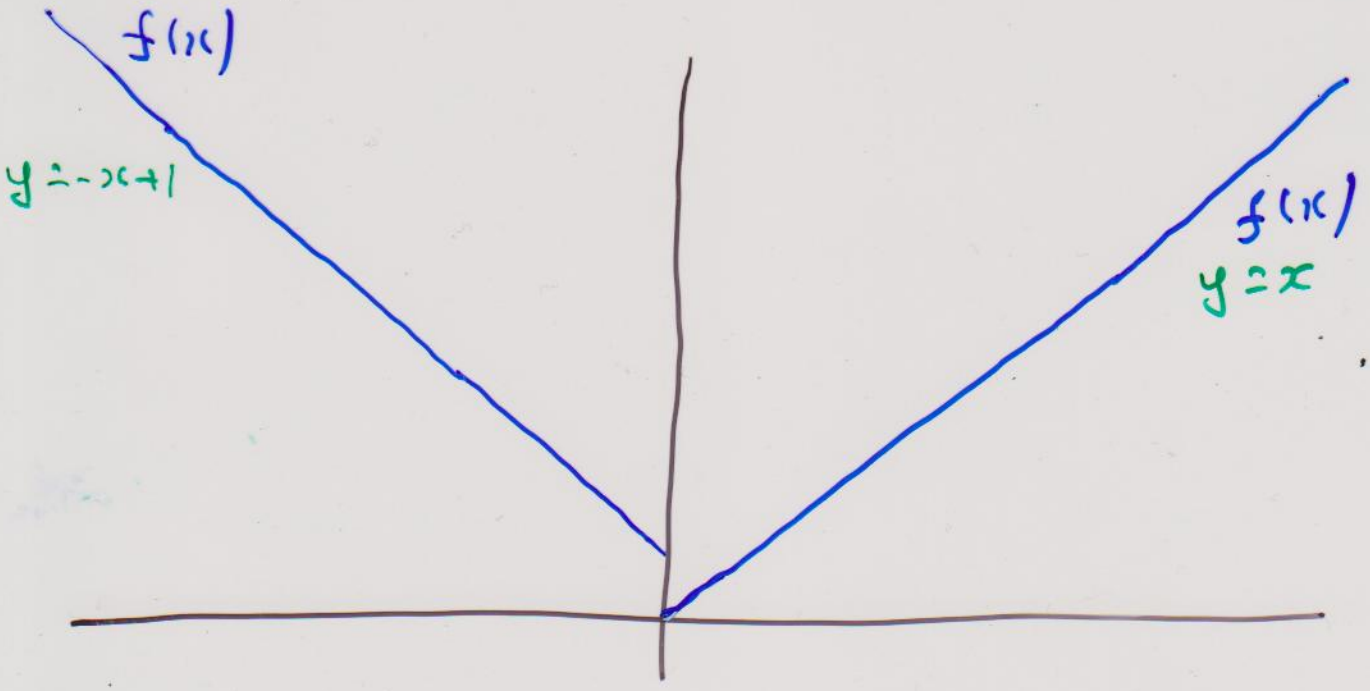
$\lim_{x \to -1} f(x) = 2 = f(-1)$

So $f(x)$ is continuous at $x = -1$.

$\lim_{x \to 0} f(x)$ does not exist.

So $\lim_{x \to 0} f(x) \neq f(0)$

Thus $f(x)$ is <u>not</u> continuous at $x = 0$.

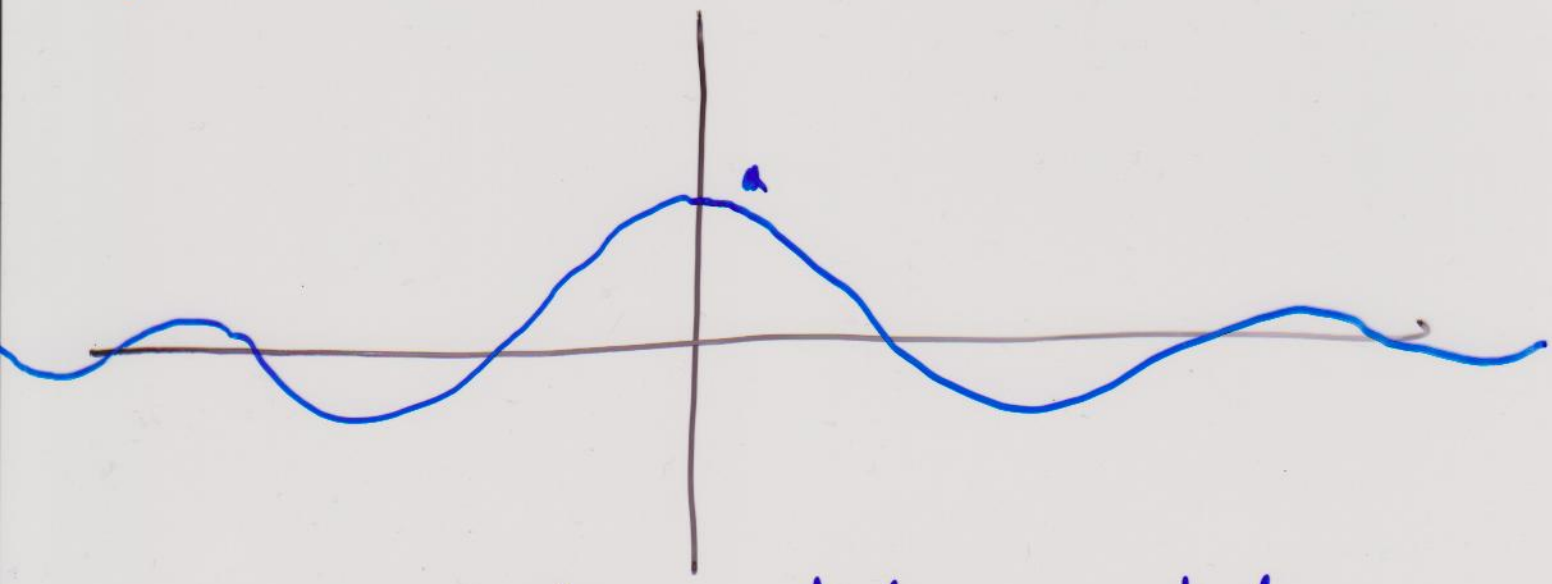


Example Consider

$$f(x) = \begin{cases} \dfrac{\sin(x)}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1 = f(0)$$

Hence $f(x)$ is continuous at $x = 0$ (and also at all other points).



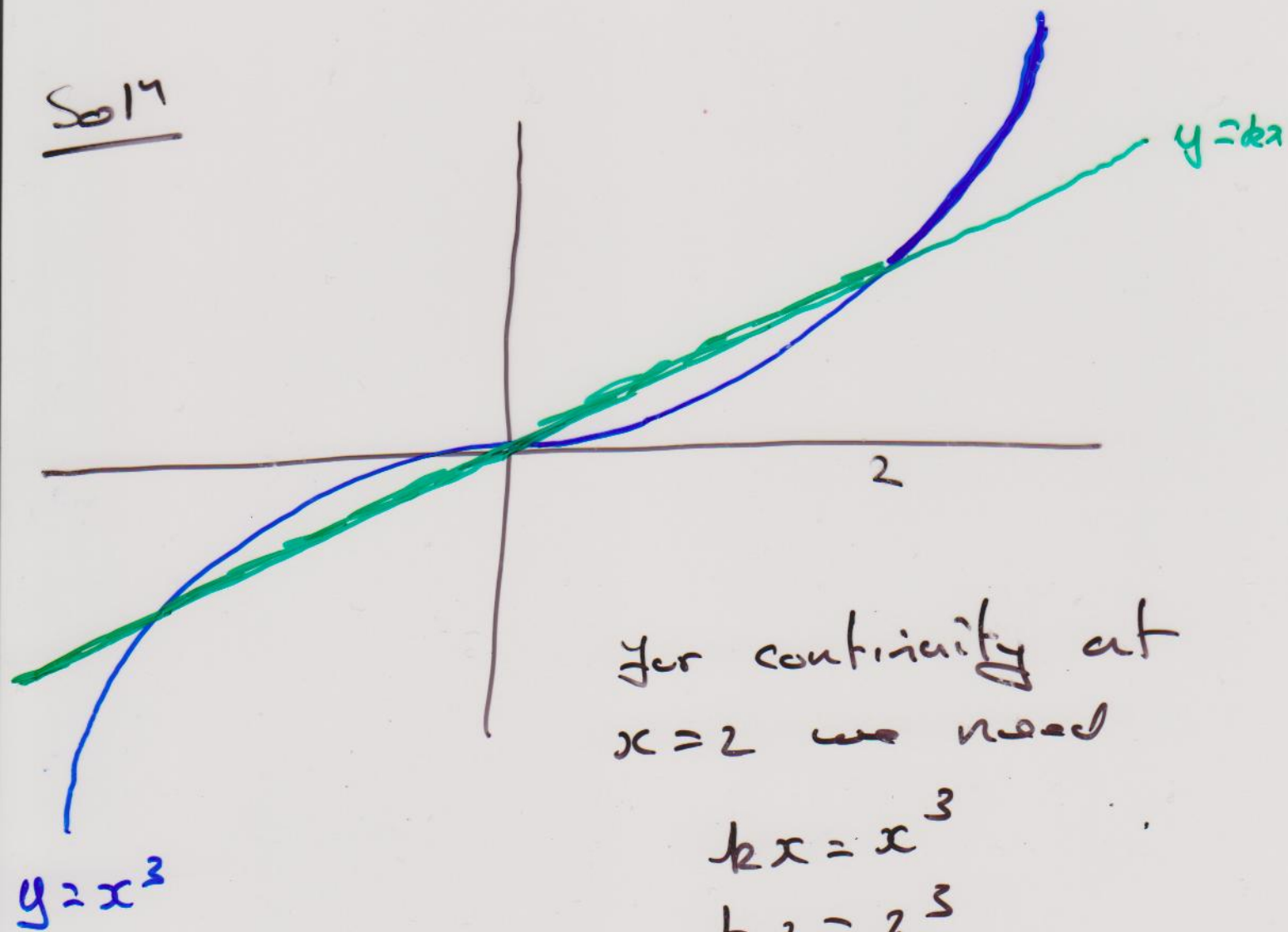
x-axis is a horizontal asymptote of $f(x)$.

Example Determine the constant k such that

$$f(x) = \begin{cases} x^3 & \text{for } x \geq 2 \\ kx & \text{for } x < 2 \end{cases}$$

is continuous at all points.

Soln



for continuity at $x = 2$ we need

$$kx = x^3$$

$$k \cdot 2 = 2^3$$

$$\boxed{k = 2^2 = 4}$$