Limits at infinity

$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

For large positive x the function $f(x)$ is approx. equal to 1. This is because

$$f(x) = \frac{\frac{x}{x}}{\frac{\sqrt{x^2+1}}{x}}$$

$$= \frac{1}{\sqrt{\frac{x^2+1}{x^2}}} = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

so

$$f(x) = \frac{1}{\sqrt{1+\frac{1}{x^2}}} \longrightarrow 1 \quad \text{as } x \to \infty$$

we write

$$\boxed{\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = 1}.$$

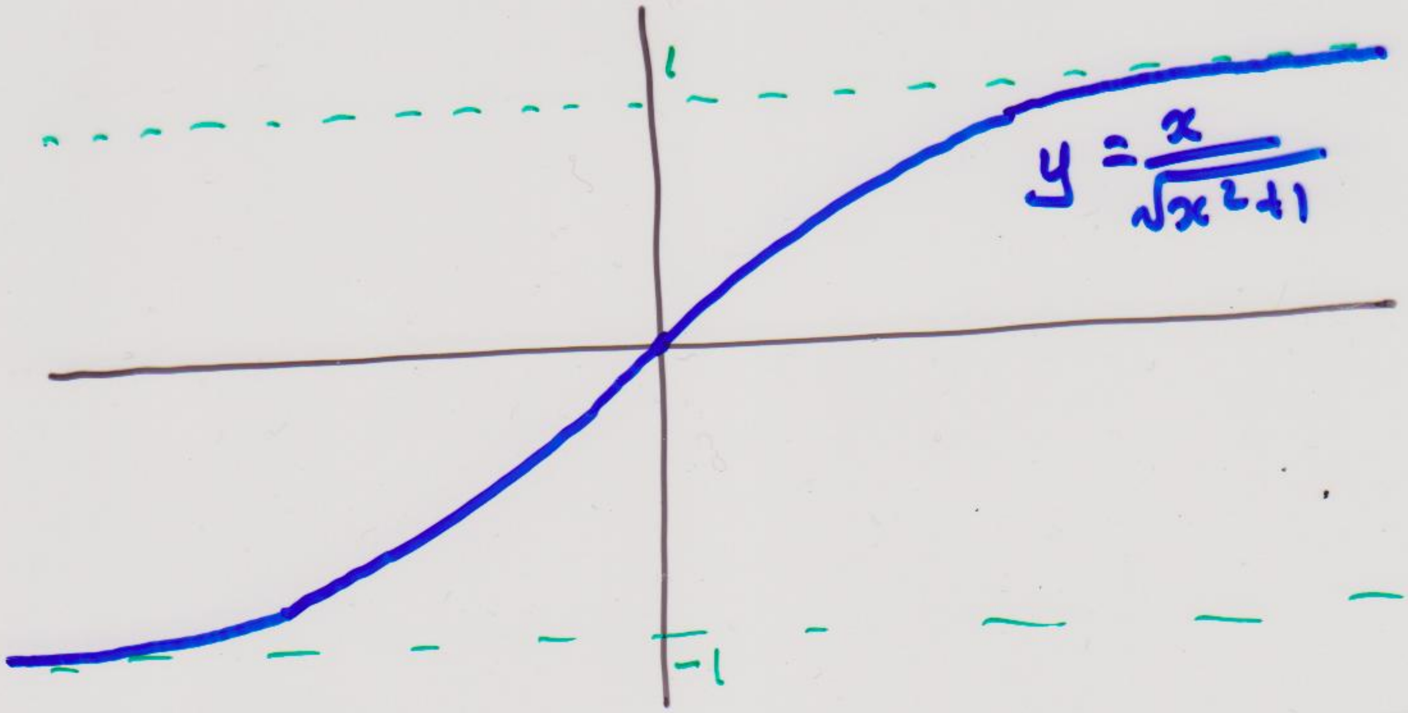
For large negative x we have that $f(x)$ is approximately equal to -1.

we write

$$\boxed{\lim_{x \to -\infty} \frac{x}{\sqrt{x^2+1}} = -1}$$

Let's sketch the graph of

$$y = \frac{x}{\sqrt{x^2+1}}.$$



The lines $y=1$ and $y=-1$ are called (horizontal) asymptotes.

Example What are the horizontal and vertical asymptotes of

$$y = \frac{2x-5}{3x+2} \quad ?$$

Then sketch the graph of y.

Sol^n

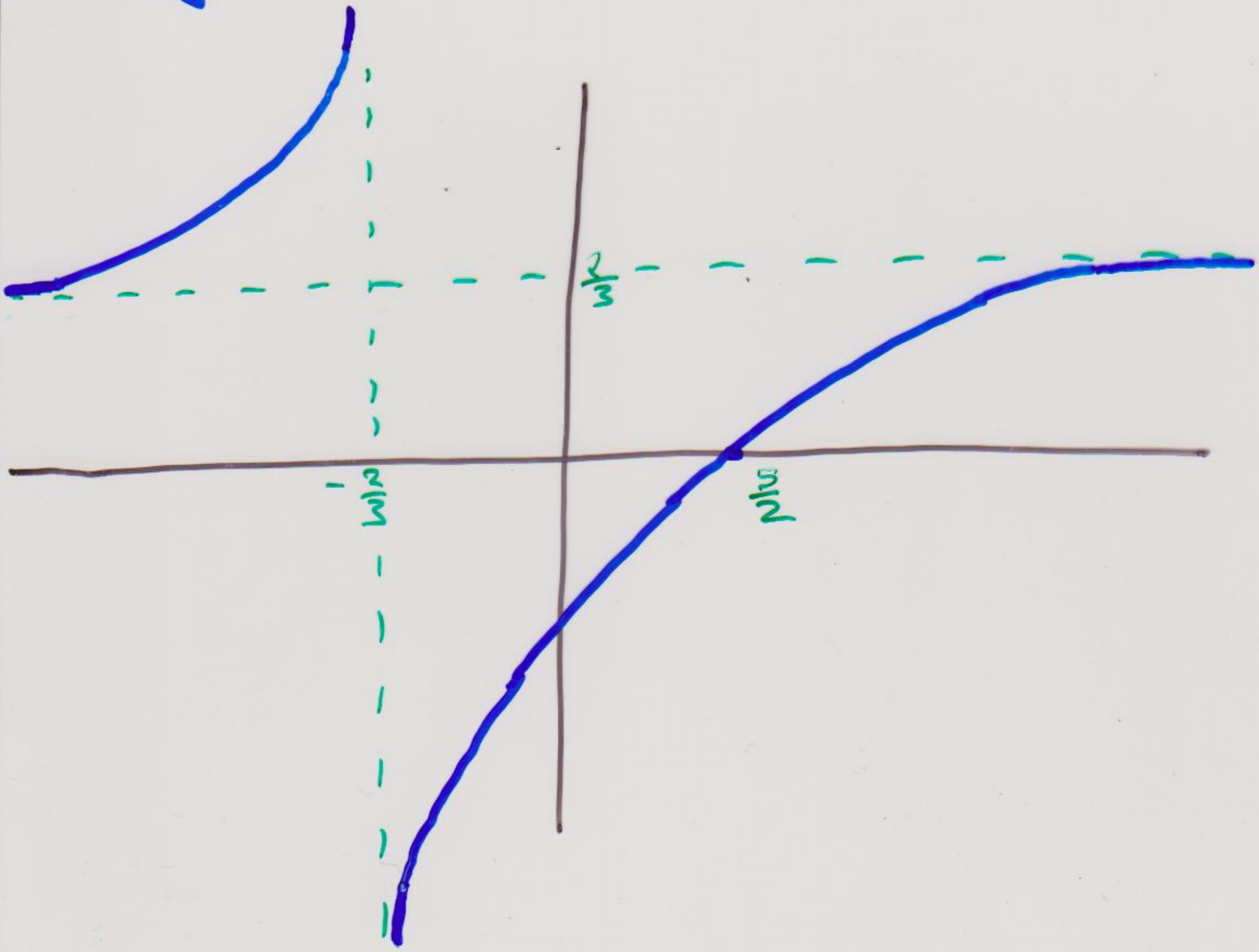
$$\lim_{x \to \infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

$$\lim_{x \to -\infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

So the line $y = \frac{2}{3}$ is a horizontal asymptote.

$\lim_{x \to -\frac{2}{3}} \frac{2x-5}{3x+2}$ does not exist.

so $x = -\frac{2}{3}$ is a vertical asymptote.



Example Evaluate

$$l = \lim_{x \to -\infty} \frac{1}{\sqrt{x^2+2x} - x}$$

Sol^n

$$l = \lim_{x \to -\infty} \frac{1}{(\sqrt{x^2+2x} - x)} \frac{(\sqrt{x^2+2x} + x)}{(\sqrt{x^2+2x} + x)}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2+2x} + x}{\cancel{x^2}+2x - \cancel{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2+2x} + x}{2x}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{x^2+2x}{x^2}} + \frac{x}{x}}{2}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{1+\frac{2}{x}} + 1}{2}$$

$$= \underline{1}$$