Consider $g(x) = \dfrac{x^6 - 1}{x - 1}$

We write

$$\lim_{x \to 1} g(x) = 6$$

to mean that for all numbers x near 1 but distinct from 1, the value of $g(x)$ is near 6.

Example Evaluate

$$\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x}$$

Sol^{n} For $x \neq 0$ we have

$$\frac{\sqrt{4+x} - 2}{x}$$

$$= \frac{\left(\sqrt{4+x} - 2\right)}{x} \frac{\left(\sqrt{4+x} + 2\right)}{\left(\sqrt{4+x} + 2\right)}$$

$$= \frac{4 + x - 4}{x\left(\sqrt{4+x} + 2\right)}$$

$$= \frac{1}{\sqrt{4+x} + 2}$$

So

$$\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0} \frac{1}{\sqrt{4+x}+2}$$

$$= \frac{1}{4}.$$

Example Evaluate

$$l = \lim_{x\to 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right).$$

Sol^n For $x \neq 2$

$$\frac{1}{x-2} - \frac{4}{x^2-4} = \frac{x+2-4}{x^2-4}$$

$$= \frac{x-2}{x^2-4}$$

$$= \frac{1}{x+2}$$

So

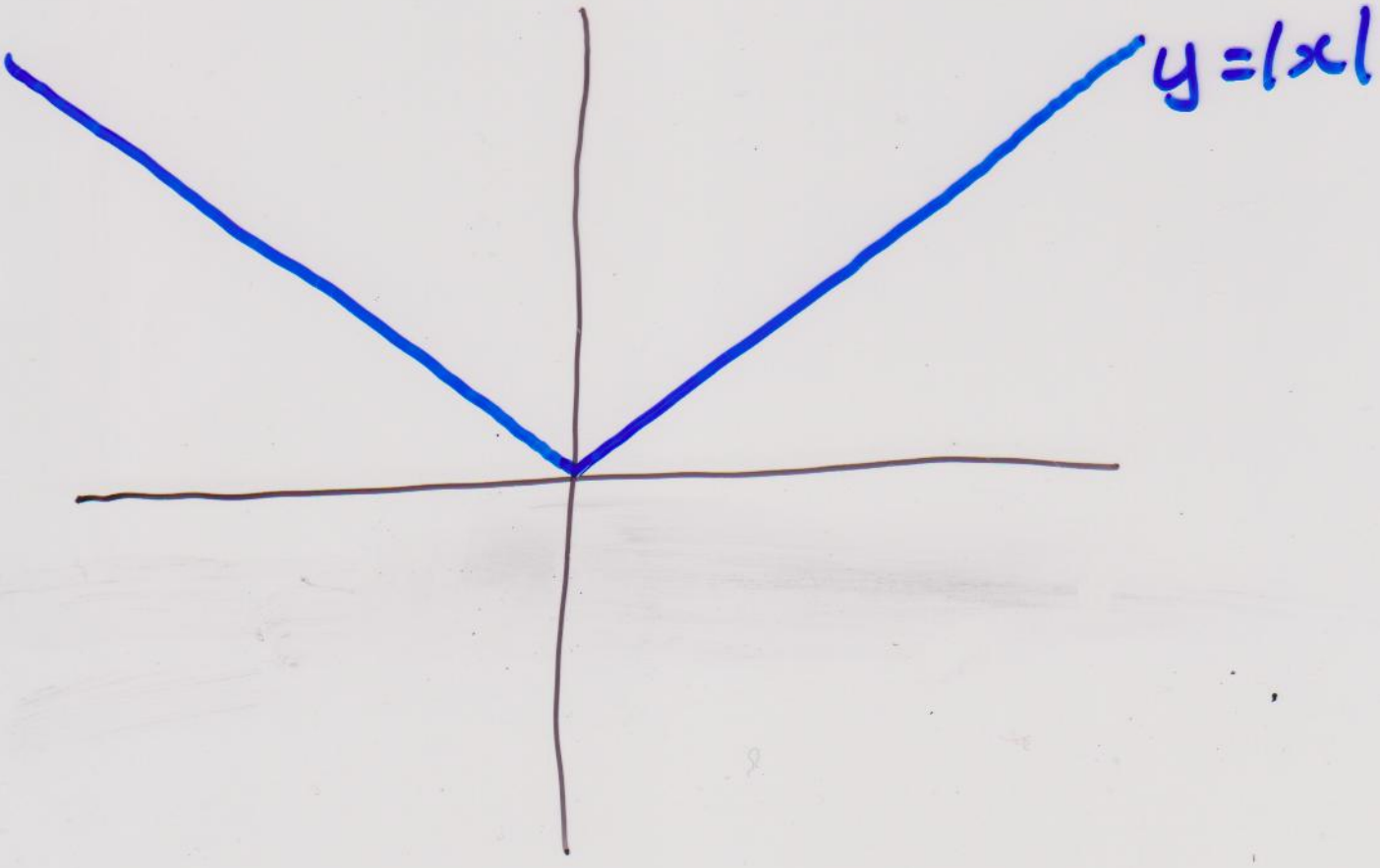
$$l = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}$$

The absolute value

$|-3| = 3$ $\qquad$ $\left|-\frac{22}{7}\right| = \frac{22}{7}$

$|2| = 2$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Example Evaluate

$$l = \lim_{x \to 0} \frac{x}{|x-1| - |x+1|}$$

Sol^n For $x \neq 0$ and x near 0

$$\frac{x}{|x-1| - |x+1|} = \frac{x}{(1-x) - (x+1)}$$

$$= \frac{x}{-2x}$$

$$= -\frac{1}{2}$$

So

$$l = \lim_{x \to 0} \left(-\frac{1}{2}\right) = -\frac{1}{2}.$$