NUI Galway
OÉ Gaillimh

Summer Examinations 2010/2011

Exam Codes	1BE1, 1BEE1, 1BEI1, 1BG1, 1BLE1, 1BM1, 1BN1 1BP1, 1BSE1, 1BV1, 1EG1
Exam	First Year Engineering
Module	Engineering Calculus
Module Code	MA140
Paper No.	1
External Examiner	Dr. C. Campbell
Internal Examiners	Dr. G. Ellis Dr. C. Röver
Instructions	Answer any SIX questions from eight
Duration	THREE HOURS
No. of Pages	Four pages
School	Mathematics, Statistics & Applied Mathematics
Requirements	Release to Library: Yes Statistical Tables: Yes - Log Tables Calculator: Yes

1. (a) Find the equation of the tangent to the curve $y = (x^2 - 5)^2$ at $x = 2$.

   (b) Show that

$$x^3 + 3x^2 - 4x - 1 = 0$$

   has three real solutions. (Hint: consider sign changes.)

   (c) Determine values of k, l, m and n such that the following function $g(x)$ is continuous and differentiable at all points.

$$g(x) = \left\{ \begin{array}{ll} 2x^2 - n & \text{if } x < -2 \\ mx + l & \text{if } -2 \leq x < 2 \\ kx^2 + 1 & \text{if } x \geq 2 \end{array} \right\}$$

2. (a) Evaluate the following.

   i. $\lim\limits_{\theta \to 0} \dfrac{\theta \sin(\theta)}{1 - \cos(\theta)}$

   ii. $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$ where $f(x) = \sqrt{x+1}$

   (b) Let

$$f(x) = x^4 - 8x^3 + 10 \ .$$

   i. Find all local maxima, all local minima and all inflection points of $f(x)$.

   ii. Determine the intervals on which $f(x)$ is decreasing.

   iii. Determine the intervals on which $f(x)$ is concave up.

   iv. Sketch the graph of $f(x)$.

3. (a) Differentiate the following.

   i. $y = \sin(x^3)$

   ii. $y = \dfrac{\sin(x)}{\sqrt{x}}$

   iii. $y = \dfrac{(x+1)\sqrt{x+2}}{(x+3)\sqrt{x+4}}$ (Hint: use logs.)

   (b) At a certain instant the length of a rectangle is 16m and the width is 12m. The width is increasing at 3m/s. How fast is the length changing if the area of the recangle is not changing?

4. (a) A box is to be made from a rectangular sheet of cardboard 70cm by 150cm by cutting equal squares out of the four corners and bending four flaps to make the sides of the box. (The box has no top.) What is the largest possible volume of the box?

   (b) A certain cell culture grows at a rate proportional to the number of cells present. If the culture contains 500 cells initially and 800 after 24 hours, how many cells will be present after a further 12 hours?

5. (a) State the Fundamental Theorem of Calculus and use it to differentiate

$$\int_{-x}^{x^3} \sqrt{t}\,\sin(t^2)\,dt$$

   with respect to x.

   (b) Evaluate the following three integrals.

$$\text{(i)} \int \frac{x^2-8x-6}{(x^2+2)(x-2)}\,dx \qquad \text{(ii)} \int_1^e \sqrt{x}\,\ln(\sqrt{x})\,dx \qquad \text{(iii)} \int_0^\infty \frac{x}{1+x^4}\,dx$$

6. (a) Use the trapezoidal method to approximate the value of

$$2\int_0^{\pi/2} \sqrt{3+\sin^2(x)}\,dx$$

   with one, two and four intervals.

   (b) Let $k \geq 0$ be an integer. Writing $\ln^k(x)$ for $(\ln(x))^k$, define

$$I_k = \int \ln^k(x)\,dx.$$

   Using integration by parts show that, for $k \geq 2$,

$$I_k = x\ln^k(x) - x\ln^{k-1}(x) - (k-1)(I_{k-1} - I_{k-2}).$$

   With this and induction, or otherwise, deduce that, for $k \geq 1$,

$$I_k = x\ln^k(x) - kI_{k-1}.$$

   Then evaluate $\displaystyle\int_1^e \ln^4(x)\,dx$.

7. (a) Sketch and calculate the bounded area enclosed by the line $y = x$ and the curve $y = \sin(\pi x/2)$.

   (b) Calculate the volume of the solid obtained by revolving the area between the graph of $y = \frac{1}{9}x\sqrt{36-x^2}$ and the positive x-axis around the x-axis.

   (c) Calculate the length of the arc given by $y = \frac{2}{3}(x-1)^{2/3}$ for $1 \leq x \leq 4$.

8. Find the general solution of each of the following differential equations.

(a) $\dfrac{dx}{dt} - 2tx = t^3$

(b) $t^2 \dfrac{dx}{dt} = x^2 + 2xt$

(c) $2\dfrac{d^2x}{dt^2} - 3\dfrac{dx}{dt} + x = 0$