Semester II Examinations 2009/2010

Exam Code(s)	1BE1, 1BEE1, 1BEI1, 1BG1, 1BM1, 1BN1, 1BP1, 1BSE1, 1BV1, 1EG1
Exam(s)	1st Engineering
Module(s)	Mathematics
Module Code(s)	MA150
Paper No	1 (Calculus)
Repeat Paper	
External Examiner(s)	Prof. D. Armitage
Internal Examiner(s)	Prof. T. Hurley Dr. G. Ellis Dr. G. Pfeiffer

Instructions: **Answer six questions.**

Duration	3 hours
No. of Pages	4 pages
Disciplines(s)	Mathematics
Course Co-ordinators(s)	

Requirements:

Statistical Tables/ Log Tables	Yes
Graph paper	Optional

1. (a) Find the equation of the tangent to the curve $y = \sqrt{x^2 + 5}$ at $x = 2$.

(b) Show that

$$x^3 - 3x + 1 = 0$$

has three real solutions. (Hint: consider sign changes.)

(c) Determine values of k and l such that the following function $g(x)$ is continuous and differentiable at all points.

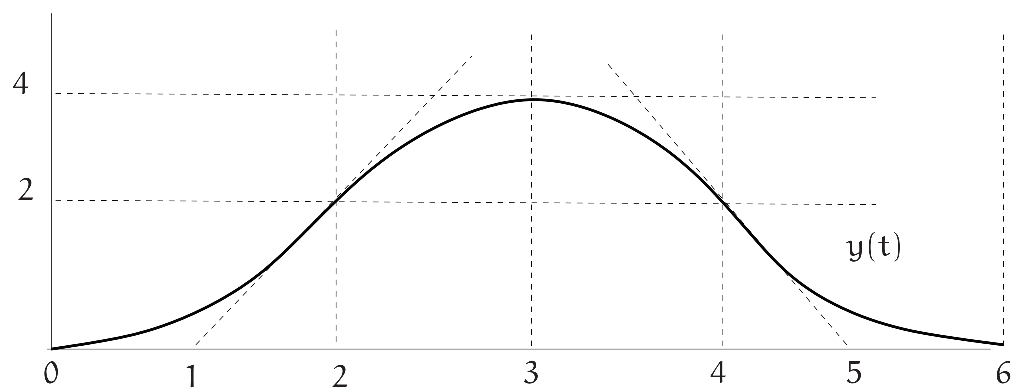
$$g(x) = \left\{ \begin{array}{ll} kx + l & \text{if } x < 2 \\ 2x^2 + k & \text{if } x \geqslant 2 \end{array} \right\}$$

2. (a) Evaluate the following.

(i) $\displaystyle\lim_{\theta \to \frac{\pi}{2}} \frac{\sin(\theta) - 1}{\theta - \frac{\pi}{2}}$

(ii) $\displaystyle\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}$ where $f(x) = x^2 + 1$

(b) A particle travels along a straight line. Its distance from a fixed point on the line at time t, where $0 \leqslant t \leqslant 6$, is a continuous function $y(t)$ whose graph is illustrated (with horizontal t-axis). There are points of inflection at $(2, 2)$ and $(4, 2)$.



(i) On which interval(s) is the particle accelerating (i.e. $y''(t) \geqslant 0$)?

(ii) On which interval(s) is the particle decelerating (i.e. $y''(t) \leqslant 0$)?

(iii) What is the maximum speed of the particle?

(iv) At what times(s) between $t = 1$ and $t = 5$ is the particle stationary?

(v) How far has the particle travelled between $t = 0$ and $t = 3$?

3. (a) Differentiate the following.

(i) $y = \cos(x^2)$

(ii) $y = \dfrac{\cos(x)}{x^3}$

(iii) $y = \dfrac{(x-1)^2\sqrt{x+2}}{(x-3)^2\sqrt{x+4}}$ (Hint: logarithmic differentiation)

(b) A rectangle with sides parallel to the x- and y-axes is inscribed in the ellipse

$$x^2 + \frac{y^2}{4} = 1.$$

Find the largest possible area for this recangle.

4. (a) An aircraft is flying horizontally at a speed of 300 km/h. How fast is the distance between the aircraft and a radio beacon increasing 1 minute after the aircraft passes 5km directly above the beacon?

(b) A cup of coffee cools at a rate proportional to the excess of its temperature above room temperature. A cup of coffee in a room at 20°C cools from 80°C to 40°C in eight minutes. How long will it take to cool to 30°C?

5. (a) State the Fundamental Theorem of Calculus and use it to differentiate

$$\int_x^{x^2} \sqrt{t}\,dt$$

with respect to x.

(b) Let $f(x) = \dfrac{1}{x^2+1}$. Estimate $\int_0^4 f(x)\,dx$ by calculating the lower Riemann sum $L(f, P_n)$ and the upper Riemann sum $U(f, P_n)$ for $n = 2$ and for $n = 4$, with respect to partitions P_n of $[0, 4]$ into n subintervals of equal length. Verify that

$$L(f, P_2) \leqslant L(f, P_4) \leqslant \tan^{-1}(4) \leqslant U(f, P_4) \leqslant U(f, P_2).$$

6. (a) Determine the following three integrals.

(i) $\displaystyle\int_0^{1/3} (1-3x)^{19}\,dx$, (ii) $\displaystyle\int \sin^3 x \cos^4 x\,dx$, (iii) $\displaystyle\int \frac{2x^3+x^2-6x+7}{x^2+x-6}\,dx$.

(b) For $n \geqslant 0$, define $I_n = \displaystyle\int x^n e^{ax}\,dx$. Use integration by parts to show that

$$I_n = \tfrac{1}{a}(x^n e^{ax} - nI_{n-1})$$

for $n \geqslant 1$ and $a \neq 0$. Compute I_0 and hence evaluate $\displaystyle\int x^3 e^{2x}\,dx$.

7. (a) Find the area A between the parabola $x = y^2 = 4x$ and the line $y = 2x - 4$.

(b) Show that the volume V generated by rotating the region bounded by the curve

$$x = \frac{1}{y-1}$$

and the lines $x = 1$, $x = 4$, and $y = 1$ about the x-axis is $(4\ln 2 + \frac{3}{2})\pi$.

(c) Find the length L of the curve $y = x^2 - \frac{1}{8}\ln x$ from $x = 1$ to $x = 2$.

8. Solve the following differential equations:

(a) $xy' + y = x^3$,

(b) $y' = \dfrac{x^2 - y^2}{2xy}$,

(c) $y'' + y' - 2y = 0$.