**Problem** Evaluate

$$I = \int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{2}}} 2x \cos(x^2)\, dx$$

**Sol$^{n}$**

Let $u = x^2$ $\qquad x = \sqrt{\frac{\pi}{4}}, \; u = \frac{\pi}{4}$

$du = 2x\,dx$ $\qquad x = \sqrt{\frac{\pi}{2}}, \; u = \frac{\pi}{2}$

$\cancel{\tfrac{1}{2}du = x\,dx}$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(u)\, du$$

$$= \sin(u) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)$$

$$= 1 - \frac{1}{\sqrt{2}} \,.$$

# Technique 4:

## Integration by parts

Recall how we differentiate a product.

$$\left(u(x)\,v(x)\right)' = u'(x)\,v(x) + u(x)\,v'(x)$$

so

$$\int \left(u(x)\,v(x)\right)' dx = \int u'(x)\,v(x)\,dx + \int u(x)\,v'(x)\,dx$$

Let's rewrite this as:

$$\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$$

= write one down
× integral of other

− integral of $\left(\begin{array}{l}\text{the one already found} \\ \times \text{ derivative of the first}\end{array}\right)$

## Problem Find

$$I = \int \underset{u}{x}\ \underset{dv}{\sin(x)}\ dx$$

Sol$^n$

$$I = \overset{u}{x}\,(\overset{v}{-\cos x}) - \int (\overset{v}{-\cos x})\ \overset{du}{dx}$$

$$= -x\cos(x) + \sin(x) + C.$$

**Problem** Find

$$I = \int \underset{u}{x} \; \underset{dv}{\sec^2(x)} \; dx$$

**Sol^n** (Recall $\frac{d}{dx} \tan x = \sec^2 x$)

$$I = x \tan x - \int \tan x \; dx$$

$$= x \tan x + \int \frac{-\sin x}{\cos x} \; dx$$

$$= x \tan x + \ln(\cos(x)) + C.$$

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**Problem** Evaluate

$$I = \int \ln(x) \; dx$$

Sol^n

$$I = \int \underset{dv}{1} \cdot \underset{u}{\ln(x)} \, dx$$

$$= x\ln(x) - \int x \frac{d}{dx}(\ln(x))\, dx$$

$$= x\ln(x) - \int \frac{x}{x}\, dx$$

$$= x\ln(x) - \int 1\, dx$$

$$= x\ln(x) - x + C.$$

Problem Evaluate

$$I = \int \underset{u}{x}\, \underset{dv}{e^x}\, dx.$$

Sol^n

$$I = x e^x - \int e^x\, dx$$

$$= x e^x - e^x + C.$$