Problem A particle is shot straight upwards from the ground with initial velocity 98 m/s. its velocity at time t is given by

$$v = -9.8t + 98$$

1) when does it reach its maximum height?

2) what is the maximum height it achieves?

Sol^n

1) Maximum height occurs when velocity is zero, so, it occurs when

$$0 = -9.8t + 98$$

So maximum height occurs at time $t = 10$ secs.

2) From last lecture we see that

$$\text{maximum height} = \int_0^{10} (-9.8t + 98)\, dt$$

$f(t) = -9.8t + 98$

anti-derivative $F(t) = -\frac{1}{2} 9.8t^2 + 98t$

$$\text{Max height} = F(10) - F(0)$$

$= -4.9 \times 100 + 98 \times 10 - 0$

$= -490 + 980$

$= \underline{\underline{490 \text{ m}}}.$

Problem The birth rate in a city t years after 1960 was $13+t$ thousands of births per year. How many births occured between 1960 and 1980?

Sol^n

Total number of births $= \int_0^{20} (13+t)\,dt$

$f(t) = 13+t$

$F(t) = 13t + \frac{1}{2}t^2$

Total number births $= F(20) - F(0)$

$= 13 \cdot 20 + \frac{1}{2}\,400$

$= 260 + 200$

$= 460$ thousand births.

<u>Problem</u> The previous city had a death rate of $5 + \frac{t}{2}$ thousands per year t years after 1960.

Assuming net immigration to the city equals net emmigration from the city, what is the increase in the population of the city by 1980.

Sol[n]

Increase in population by 1980 $= \displaystyle\int_0^{20} 13 + t - \left(5 + \frac{t}{2}\right)$

$$= \int_0^{20} \left(8 + \frac{t}{2}\right) dt$$

$$= 8t + \frac{t^2}{4} \Big|_0^{20}$$

$$= 160 + 100$$

$= 260$ thousands

Problem The rate of flow of water into an initially empty tank is $100 - 3t$ gallons per minute at time t minutes. How much water flows into the tank between $t = 10$ and $t = 20$ minutes?

Sol^n

you try it!