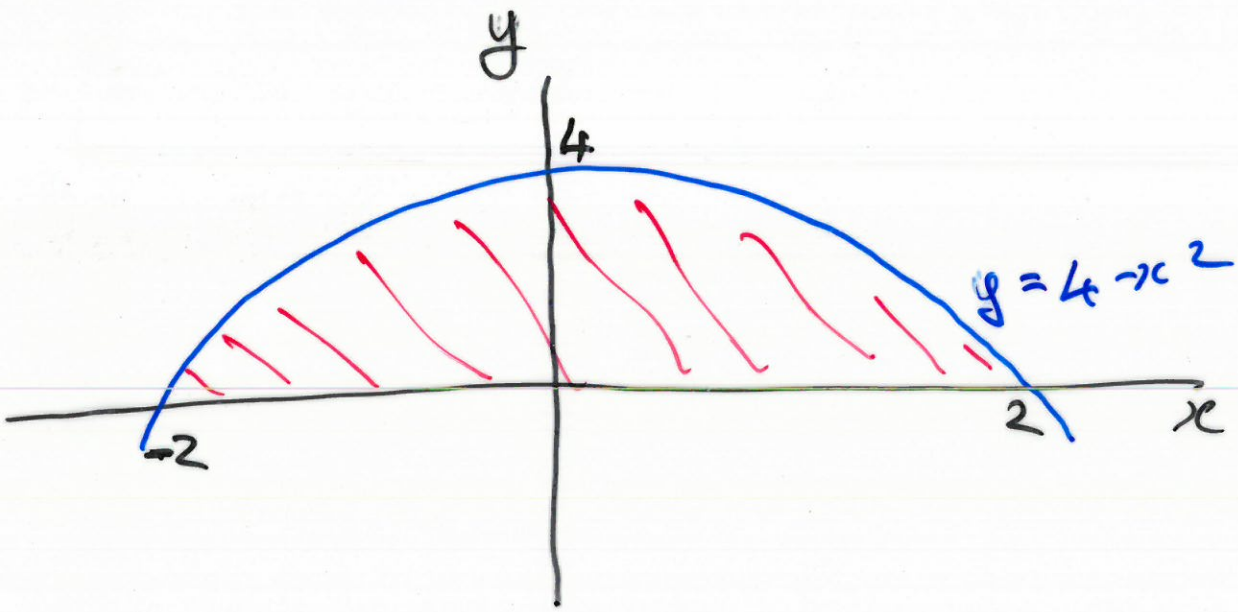
Tutorials:

Wed 3pm IT-203 (MA160)

Thur 5pm ADB-1020 (MA135)

Problem Find the area of the region bounded by $y = 4 - x^2$ and the x-axis.

Step 1 Draw a picture of the region.



So the required area is

$$I = \int_{-2}^{2} (4 - x^2)\, dx$$

since $y = 0$ for $x = -2$ and $x = 2$.

Step 2 Use the following:

Fundamental Theorem of Calculus

If $f(x) = \frac{d}{dx} F(x)$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

In our problem

$$f(x) = 4 - x^2.$$

An anti-derivative is

$$F(x) = 4x - \frac{1}{3}x^3$$

so required area is

$$I = \int_{-2}^{2} (4 - x^2)\,dx = F(2) - F(-2)$$

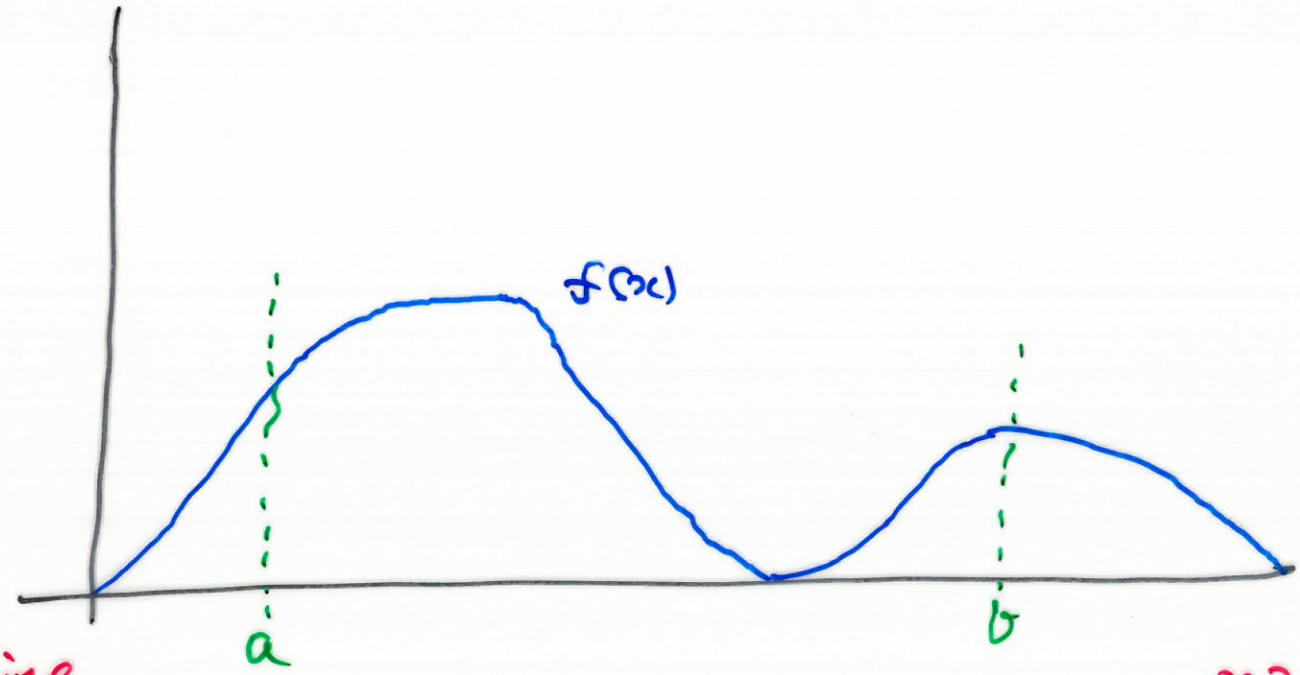
$$= \left(4.2 - \tfrac{1}{3}2^3\right) - \left(-4.2 + \tfrac{1}{3}2^3\right)$$

$$= 16 - \frac{16}{3} = \frac{32}{3}.$$

Explanation of the Fundamental Theorem of Calculus

Imagine that a train is travelling with speed $f(x)$ at time x.

Maybe we have



time $x=0$ at Galway

$x=t$ in Dublin

Suppose that the mileometer is broken on the train, but that the speedometer is working fine, and that the driver has an accurate watch.

The driver wants to approximate the distance travelled between time $x = a$ and time $x = b$.

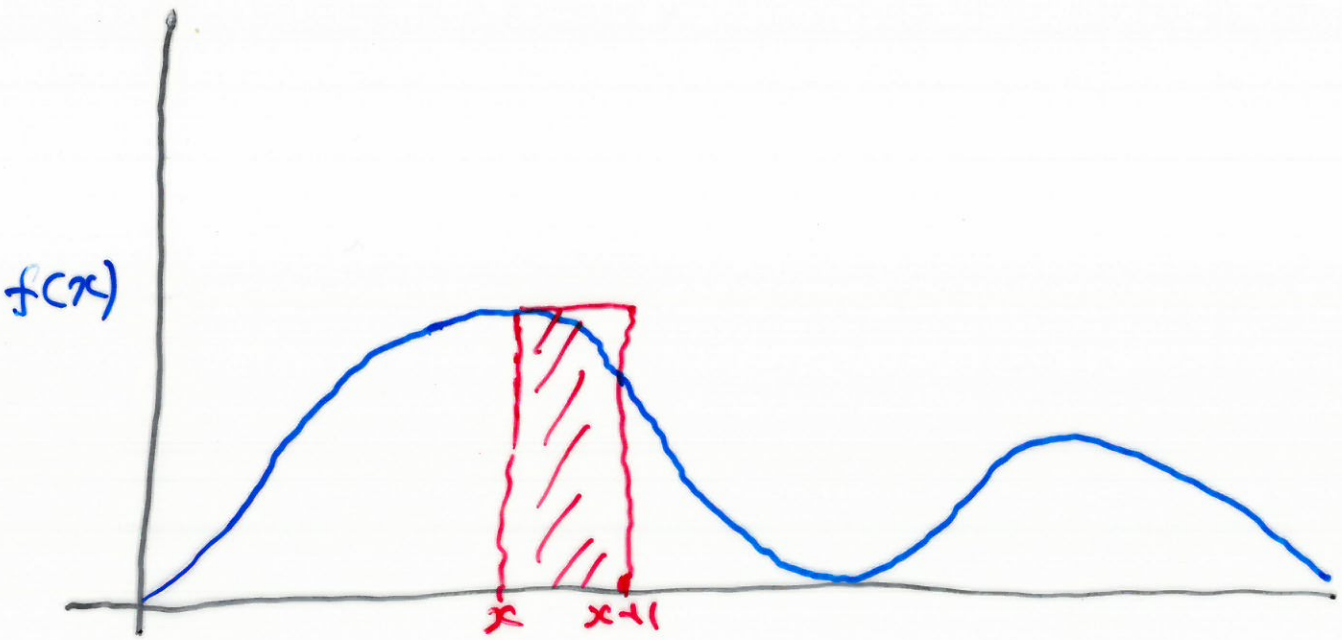
Question: How can she do this?

Answer: Measure the speed every minute, and assume that the speed is roughly constant over each minute interval.

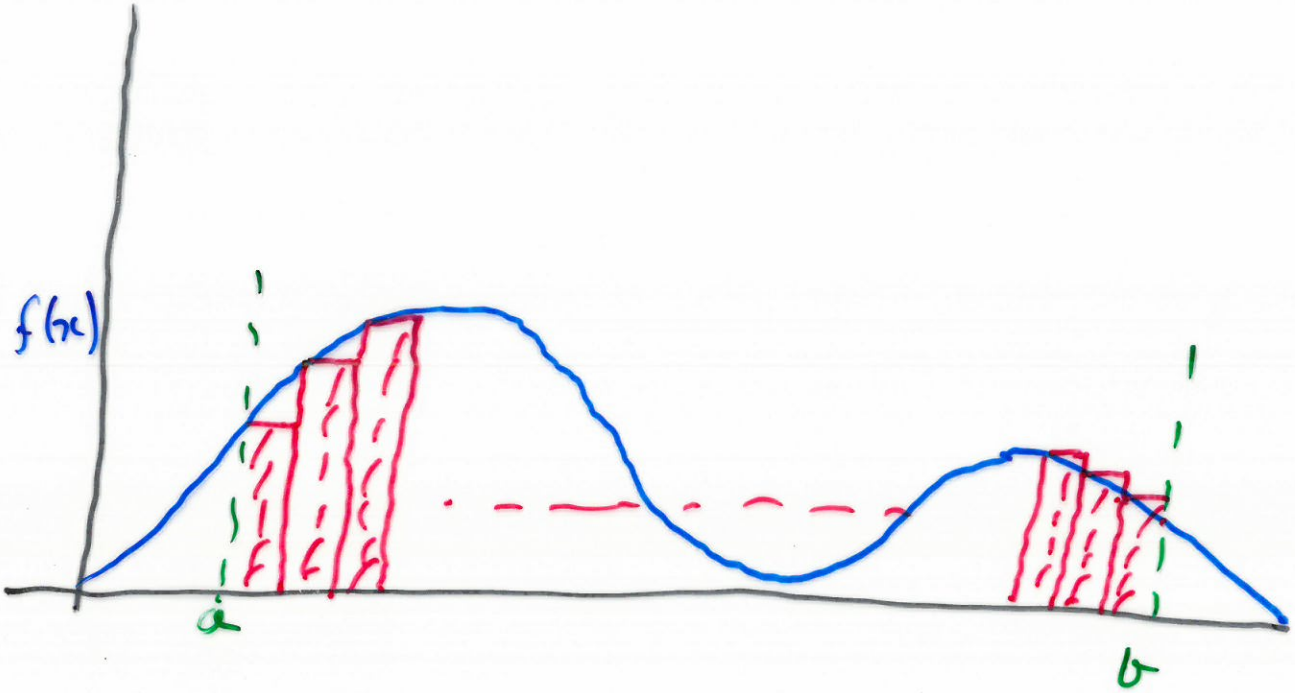
Distance travelled between x and $x+1$ minutes is roughly equal to

speed $\times$ time $= f(x) \times 1$

$=$ area of red box below.



So the total time travelled from time $x = a$ to $x = b$ is roughly



equal to:

Distance travelled $\approx$ area of red boxes $\approx$ area between curve $y = f(x)$ and the x-axis.

$$= \int_a^b f(x)\, dx \quad (1)$$

$f(x)$ = speed = rate of change of distance at time x.

What function $F(x)$ has

$$\frac{d}{dx} F(x) = f(x) \quad ?$$

Answer: $F(x)$ = distance travelled by time x.

So the distance travelled between time $x = a$ and $x = b$ is $F(b) - F(a)$. (2)

From (1) and (2) we get

$$\int_a^b f(x)\,dx \simeq F(b) - F(a).$$

The approximation becomes more accurate on taking smaller time intervals. In the limit, as the time intervals tend to 0, we get

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Name	Student Id	MH100	MH155
Sarah Skeffington	16421724		✓
Noel Minogue	16391381	✓	
Lorcan Creedon	16422524	✓	
Francie Johnston	15260501		
Sarah Rellington	15104599	✓	
Odette oconnor	14103995	✓	
Colin Roche	16402224	✓	
Chris mullins	16406252	✓	
John Cummins	16373731	✓	
Marienella Gallo	16345993	✓	
Joan Rohan	15104654	✓	
Daniel McFadden	16280010	✓	
David Bryan	16381003	✓	
Emma Kenny	16455696	✓	
Habib Sanusi	15104951	✓	
Seán McCann	16343643	✓	
David Tubridy	15104955	✓	
Sinéad O Sullivan	16385673		✓
Sarah conwell	16354341		✓
Niamh Gallagher	16354293		✓
Ronan Furey	16354343		✓
Olivia Sweeney	16395243		✓
Sophia Cogan	16366736		✓
Conor Hanlon	16444524	✓	
Aaron Mcdermott	16492186	✓	
Andrius Stroys	16319273	✓	
Jacob Duggan	16408982	✓	
Aoife Downey	16398986	✓	
James Rourke-Murphy	15104984	✓	
David Byrne	16407376	✓	
Alan Flaherty	14104205	✓	
Ellen Nihill	16430702		✓
Anna Monaghan	16310033		✓
Rachel Giles	16479176		✓

Name	Student ID	MA160	MA135
Gabrielle Murphy	16317441		✓
Karen Healy	16403697		✓
michaela regan	16449912		✓
Jack Quane		✓	
Jordan Coyne	16409296	✓	
Karen Mather	16430216		✓
Alaine Morris	16452166		✓
Emma Ryan	16335406		✓
Shane Davis	16385423		✓
Aoife Tully	16314966		✓
emhearhartain	16440762		✓
Evan Murray	16486102	✓	