The **floor function** is the function

$$\lfloor x \rfloor = \text{greatest integer} \leq x.$$

so

$$\left\lfloor \frac{3}{2} \right\rfloor = 1$$

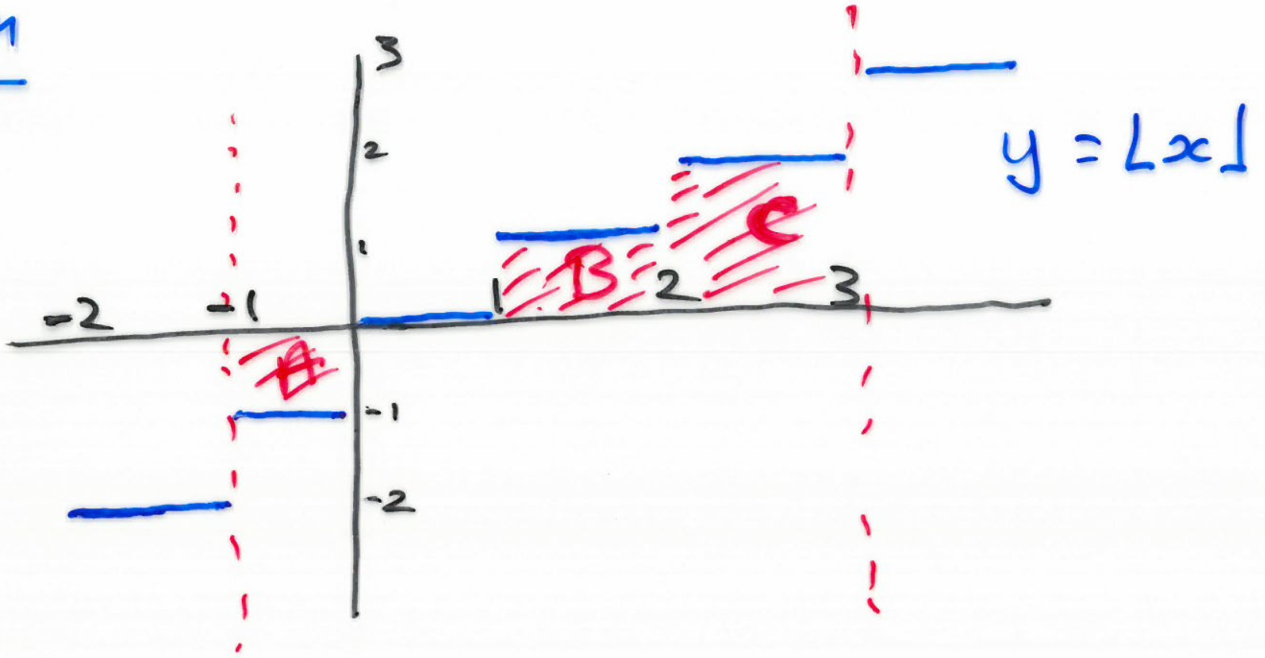
$$\lfloor -3\cdot 142 \rfloor = -4$$

$$\lfloor 3\cdot 142 \rfloor = 3$$

<u>Problem</u> Evaluate

$$\int_{-1}^{3} \lfloor x \rfloor \, dx$$

Sol^n



So $$\int_{-1}^{3} \lfloor x \rfloor \, dx = -A + B + C$$

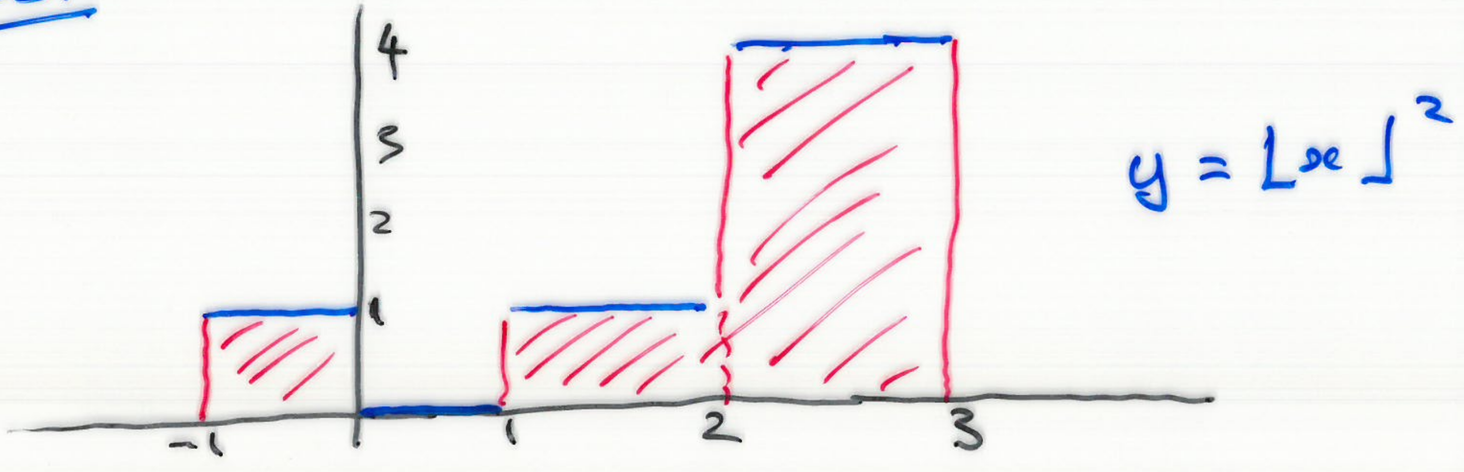
$$= -1 + 1 + 2$$

$$= 2$$

Problem Evaluate

$$\int_{-1}^{3} \lfloor x \rfloor^2 \, dx$$

Sol^n



So

$$\int_{-1}^{3} \lfloor x \rfloor^2 = 1 + 0 + 1 + 4$$
$$= 6$$

A useful result.

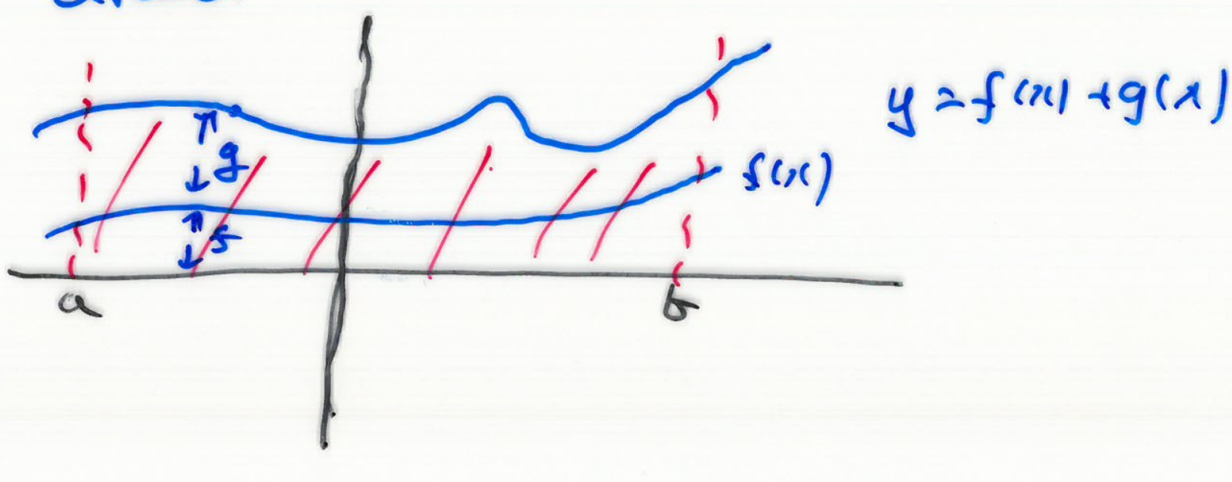
<u>Proposition 1</u>: Given two functions f, g then

$$\int_a^b (f+g)\,dx = \int_a^b f\,dx + \int_a^b g\,dx$$

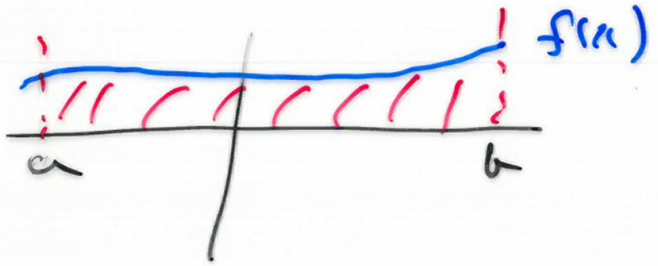
(for "nice" f, g).

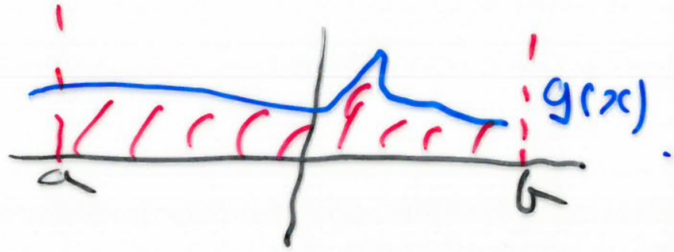
<u>Explanation</u>

The area



is the sum of the two areas



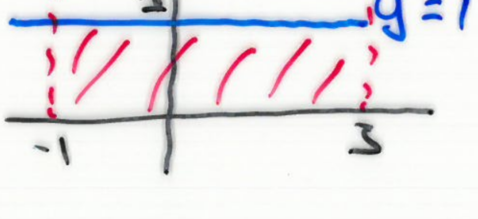


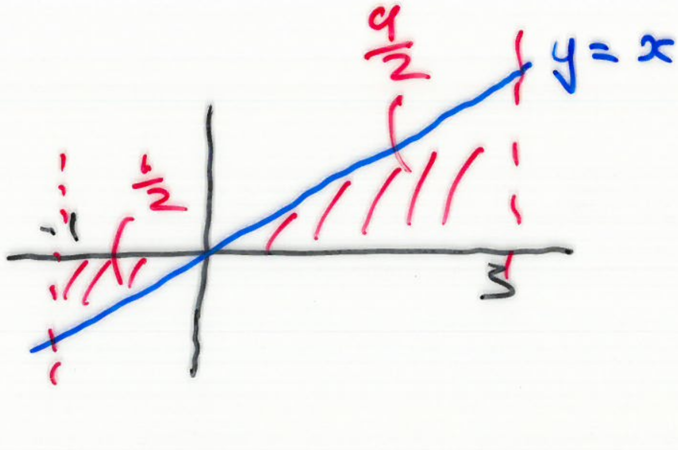
Problem Evaluate

$$I = \int_{-1}^{3} (1 + x + \lfloor x \rfloor^2) \; dx$$

Sol^n By Proposition 1

$$I = \int_{-1}^{3} 1 \, dx + \int_{-1}^{3} x \, dx + \int_{-1}^{3} \lfloor x \rfloor^2 \, dx$$





$$I = 4 + \left(-\tfrac{1}{2} + \tfrac{9}{2}\right) + 6$$

$$= 14$$

Definition We say that a function $F(x)$ is an anti-derivative of a function $f(x)$ if

$$\frac{d}{dx} F(x) = f(x).$$

Problem Find an anti-derivative of $f(x) = x + 1$.

Sol^n $F(x) = \frac{1}{2}x^2 + x$

or $F(x) = \frac{1}{2}x^2 + x + 5$

Problem Find an anti-derivative of $f(x) = 2 + 3x^2 + \sin(x)$.

Sol^n $F(x) = 2x + x^3 - \cos(x)$

or $F(x) = 2x + x^3 - \cos(x) + 7$

Problem Find an anti-derivative of $f(x) = x^n$.

Sol^n $F(x) = \frac{1}{n+1} x^{n+1}$.

or $F(x) = \frac{1}{n+1} x^{n+1} + 42$

Fundamental Theorem of Calculus

Suppose that $F(x)$ is an anti-derivative of a "nice" function $f(x)$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$