# MA121, 2005 Q7

a) (i) Solve

$$\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}, \qquad y(0) = \pi.$$

Sol^n

$$(2y + \cos y)\frac{dy}{dx} = 6x^2$$

$$\int 2y + \cos y \; dy = \int 6x^2 \, dx$$

$$y^2 + \sin y = 2x^3 + c$$

$y(0) = \pi$ :

$$\pi^2 + \sin \pi = 0 + c$$

$$\pi^2 = c$$

Solution is :

$$\boxed{y^2 + \sin y = 2x^3 + \pi^2}$$

a) ii) Solve

$$\frac{dy}{dx} + 2xy = x e^{-x^2}, \quad y(0) = 2.$$

Sol^n

Equation is first order linear.

We'll multiply both sides by $e^{\int 2x \, dx}$.

$$e^{\int 2x \, dx} = e^{x^2}.$$

$$e^{x^2} \frac{dy}{dx} + e^{x^2} 2xy = e^{x^2} x e^{-x^2}$$

$$e^{x^2} \frac{dy}{dx} + e^{x^2} 2xy = x$$

$$\frac{d}{dx}\left( e^{x^2} y \right) = x$$

$$e^{x^2} y = \int x \, dx$$

$$e^{x^2} y = \frac{x^2}{2} + c$$

$y(0) = 2$ :

$$e^0 \cdot 2 = 0 + c$$

$$c = 2$$

Solution is

$$e^{x^2} y = \frac{x^2}{2} + 2$$

$$\boxed{y = e^{-x^2}\left(\frac{x^2}{2} + 2\right).}$$

Q7 b)

A radioactive substance decays according to the law

$$\frac{dy}{dt} = -ky$$

where $y(t)$ is the amount of the substance present at time $t$, and $k > 0$ is a constant. Determine the half life of the substance.

Sol^n Suppose

$$y(0) = y_0$$

we need to find the value of $t$ such that

$$y(t) = \frac{1}{2} y_0 .$$

The solution to the diff. eq^n. is of the form

$$y(t) = A e^{-kt}$$

~~Need to solve~~

$$y_0 = y(0) = A$$

so $$y(t) = y_0 e^{-kt}$$

Need to solve

$$y(t) = \frac{1}{2} y_0$$

$$\cancel{y_0} e^{-kt} = \frac{1}{2} \cancel{y_0}$$

$$e^{-kt} = \frac{1}{2}$$

$$\ln(e^{-kt}) = \ln(\tfrac{1}{2})$$

$$-kt = \ln(\tfrac{1}{2})$$

half life is

$$t = -\frac{\ln(\frac{1}{2})}{k}.$$