

2015/16 Exam, MATHS-160

Q3 b) Solve the differential equation

$$\frac{dy}{dx} = \frac{4-y^2}{2xy-3y}, \quad y(1) = 0.$$

Soln

$$\frac{dy}{dx} = \frac{4-y^2}{(2x-3)y}$$

$$\frac{y}{4-y^2} \frac{dy}{dx} = \frac{1}{2x-3}$$

$$\int \frac{y}{4-y^2} dy = \int \frac{1}{2x-3} dx$$

$$-\frac{1}{2} \int \frac{-2y}{4-y^2} dy = \frac{1}{2} \int \frac{2}{2x-3} dx$$

$$-\frac{1}{2} \ln|4-y^2| = \frac{1}{2} \ln|2x-3| + C$$

$$-\ln|4-y^2| = \ln|2x-3| + C$$

$$y(1) = 0$$

$$-\ln|4| = \ln|1| + C$$

$$-\ln|4| = 0 + C$$

So

$$-\ln|4-y^2| = \ln|2x-3| - \ln 4$$

MATH 1, Exam 2005

Q 5 b) Determine

$$I = \int \frac{5x}{5+2x^2} dx$$

Soln

$$I = \frac{5}{4} \int \frac{4x}{5+2x^2} dx$$

$$= \frac{5}{4} \ln |5+2x^2| + C$$

Q 5 c) Determine

$$I = \int \frac{\ln x}{x^2} dx$$

Soln

$$I = \int \frac{1}{x^2} \overset{dv}{\ln x} \overset{u}{dx} dx$$

$$I = (\ln x) (-x^{-1}) + \int x^{-1} \cdot \frac{1}{x} dx$$

$$I = -\frac{\ln(x)}{x} + \int x^{-2} dx$$

$$I = -\frac{\ln(x)}{x} - x^{-1} + C$$

$$= -\frac{\ln(x)}{x} - \frac{1}{x} + C.$$