

Q2 c) Determine the indefinite integral

$$I = \int \frac{4x}{(x-1)^2(x+1)} dx$$

Soln Use partial fractions,

$$\frac{4x}{(x-1)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$= \frac{A(x-1)^2 + B(x+1)(x-1) + C(x+1)}{(x+1)(x-1)^2}$$

$$4x = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$x=1: 4 = 2C \Rightarrow \boxed{C=2}$$

$$x=-1: -4 = 4A \Rightarrow \boxed{A=-1}$$

$$x=0: 0 = -1 - B + 2 \Rightarrow \boxed{B=1}$$

$$I = -\int \frac{1}{x+1} dx + \int \frac{1}{x-1} dx + 2 \int \frac{1}{(x-1)^2} dx$$

$$= -\ln|x+1| + \ln|x-1| + 2 \int \frac{1}{(x-1)^2} dx$$

$$\text{Let } u = x-1$$

$$du = dx$$

$$I = \ln \left| \frac{x-1}{x+1} \right| + 2 \int \frac{1}{u^2} du$$

$$I = \ln \left| \frac{x-1}{x+1} \right| + 2 \left( -\frac{1}{u} \right) + C$$

$$I = \ln \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1} + C$$

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Q 5) a) Determine

$$I = \int \frac{x^4}{(x+1)^3} dx$$

Sol<sup>n</sup>

Let  $u = x+1, x = u-1$

$du = dx$

$$I = \int \frac{(u-1)^4}{u^3} du$$

$$= \int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^3} du$$

$$= \int u - 4 + 6\frac{1}{u} - 4u^{-2} + u^{-3} du$$

$$= \frac{1}{2}u^2 - 4u + 6\ln|u| + 4u^{-1} - \frac{1}{2}u^{-2} + C$$



$$= \frac{1}{2}(x+1)^2 - 4(x+1) + 6 \ln|x+1|$$

$$+ \frac{4}{x+1} - \frac{1}{2(x+1)} + C .$$