On the moon the acceleration due to gravity is $1.6\ m/sec^2$. A rock is dropped into a crevasse and falls 80 m before hitting the bottom. How long does the rock take to reach the bottom? How fast is it travelling when it hits the bottom?

Soln

$v(t)$ = velocity at time $t$.

$a(t)$ = acceleration at time $t$.

$$a(t) = 1.6$$

$$v(t) = \int_0^t a(t)\,dt = \int_0^t 1.6\,dt = 1.6t$$

$y(t)$ = distance at time $t$.

$$y(t) = \int_0^t v(t)\,dt = \int_0^t 1.6t\,dt = 0.8t^2$$

Required to find the time $t$ for which $y(t) = 80$.

$$0.8t^2 = 80$$

$$t^2 = 100$$

$$\boxed{t = 10}$$ ← time to reach bottom

$$v(10) = 1.6 \times 10 = \boxed{16 \text{ m/sec}}.$$

↑ speed at bottom.

Do not write in red on exam papers!!

MA135 Exam 2015-16

Q2 a)

Determine the indefinite integral

$$I = \int \underset{u}{x^2} \underset{dv}{e^x}\, dx$$

Soln

$$I = x^2 e^x - 2\int \underset{dv}{e^x}\, \underset{u}{x}\, dx$$

$$I = x^2 e^x - 2\left( x e^x - \int e^x\, dx \right)$$

$$I = x^2 e^x - 2xe^x + 2e^x + C.$$

Q 2b) Determine

$$I = \int \frac{\sqrt{9-x^2}}{x^2}\, dx$$

Soln

Let $x = 3\sin\theta$

$$dx = 3\cos\theta\, d\theta$$

$$I = \int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta}\, 3\cos\theta\, d\theta$$

$$= \int \frac{\sqrt{9}\sqrt{1-\sin^2\theta}}{9\sin^2\theta}\, 3\cos\theta\, d\theta$$

$$= \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \int \frac{1-\sin^2\theta}{\sin^2\theta}\, d\theta$$

$$= \int \frac{1}{\sin^2\theta} - 1\, d\theta$$

$$= \int \operatorname{cosec}^2\theta \, d\theta - \theta$$

$$= -\cot\theta - \theta + C .$$