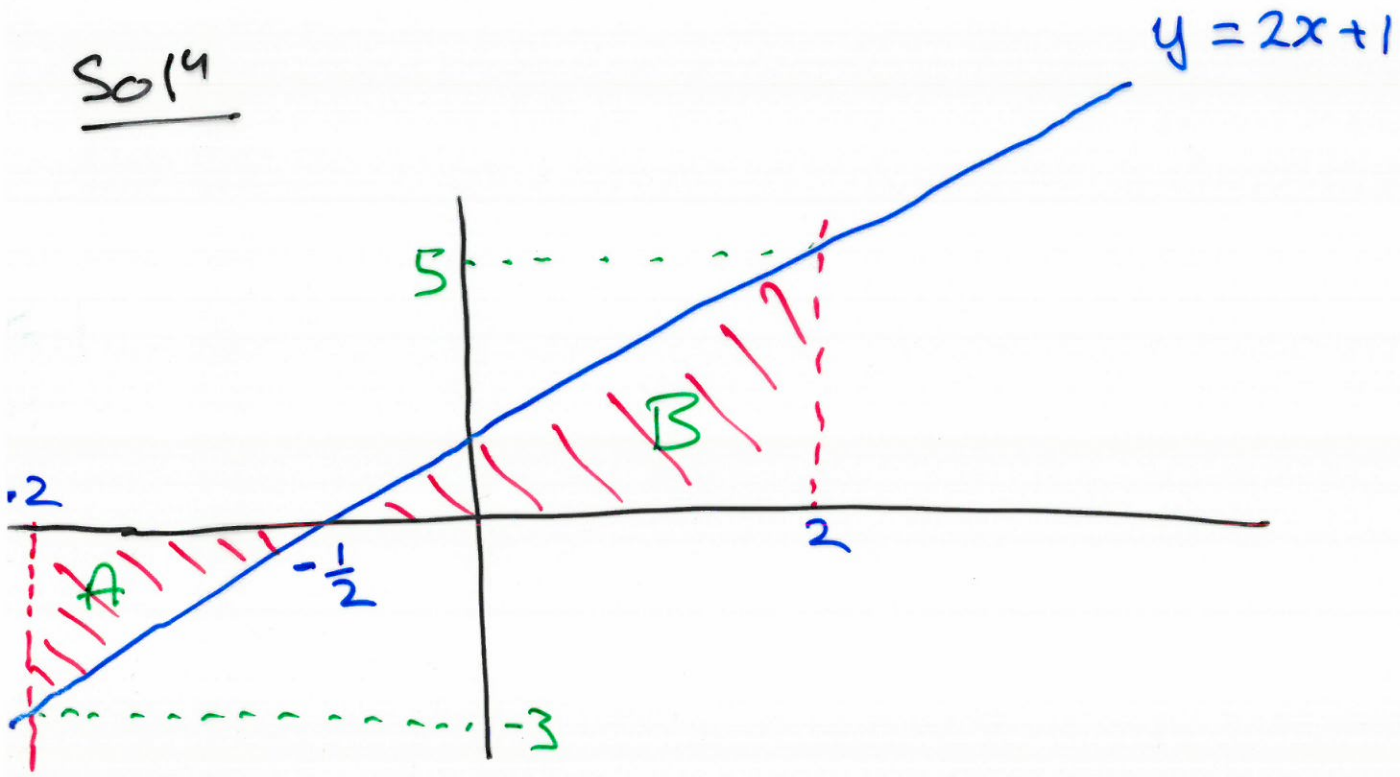
Example Evaluate

$$\int_{-2}^{2} 2x+1 \, dx .$$

Soln



area of A $= \frac{1}{2} \cdot \frac{3}{2} \, 3 = \frac{9}{4}$

area of B $= \frac{1}{2} \cdot \frac{5}{2} \, 5 = \frac{25}{4}$

so

$$\int_{-2}^{2} 2x+1 \, dx = \frac{25}{4} - \frac{9}{4} = 4 .$$

Absolute value function

Define

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & x < 0 \end{cases}$$

Example

$$|-5| = 5$$

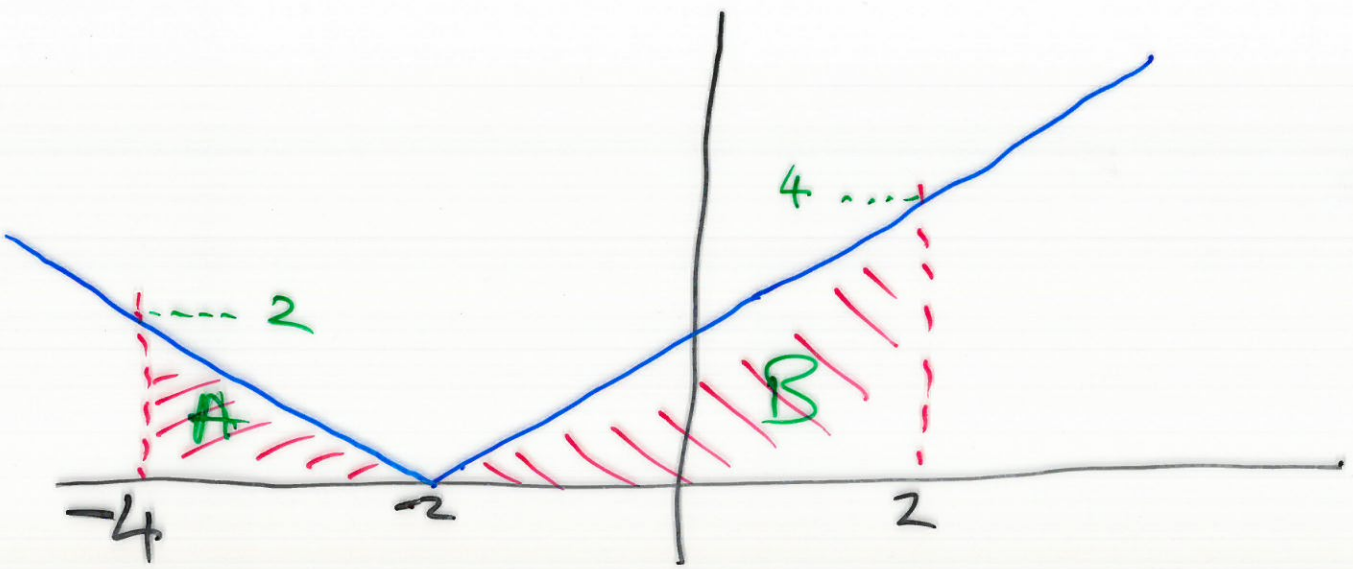
$$|5| = 5$$

$$\left|-\frac{1}{2}\right| = \frac{1}{2}$$

Example Evaluate

$$\int_{-4}^{2} |x+2| \, dx$$

Soln



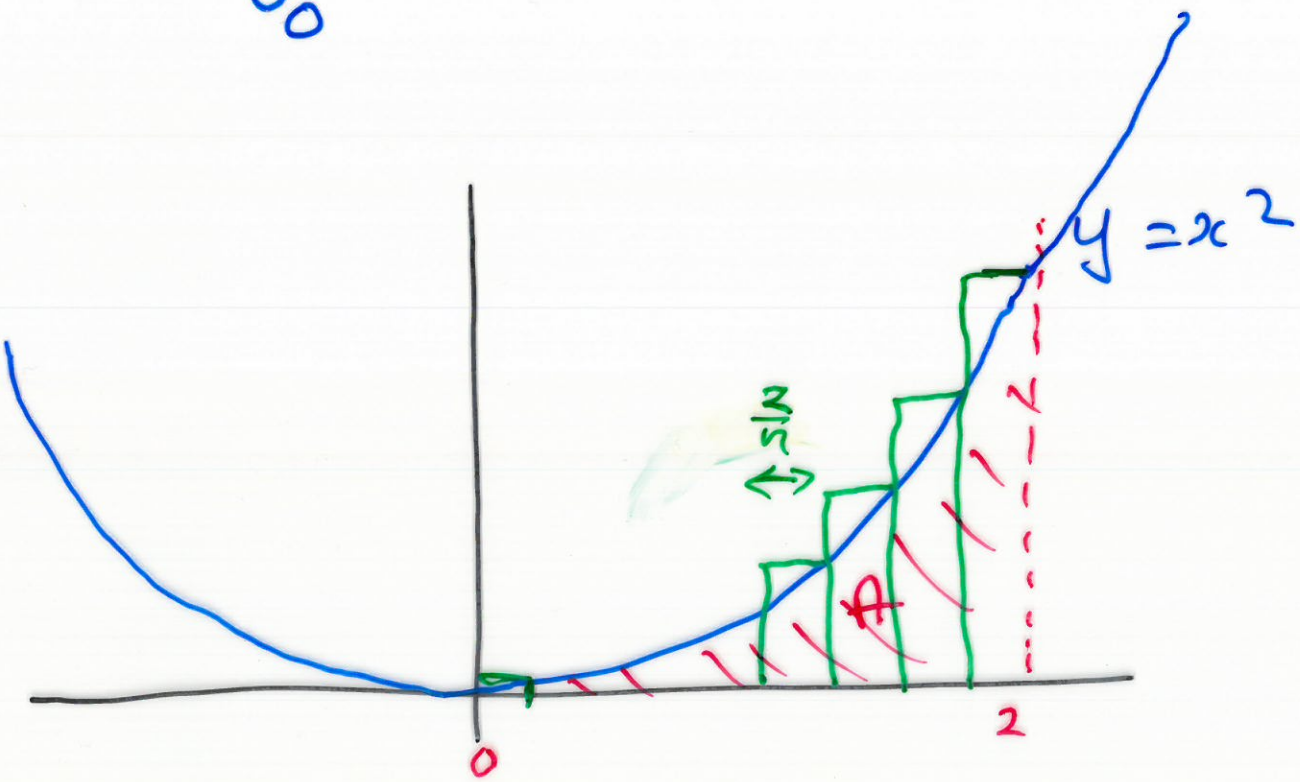
area of A $= \frac{1}{2} \cdot 2 \cdot 2 = 2$

area of B $= \frac{1}{2} \cdot 4 \cdot 4 = 8$

$$\int_{-4}^{2} |x+2| \, dx = 2 + 8 = 10 .$$

How about

$$\int_{0}^{2} x^2 \, dx \quad ?$$



n rectangles of equal width

Let's approximate the integral using $n = 5$ rectangles.

$$A \simeq \frac{2}{5}\left(\frac{2}{5}\right)^2 + \frac{2}{5}\left(\frac{4}{5}\right)^2 + \frac{2}{5}\left(\frac{6}{5}\right)^2 + \frac{2}{5}\left(\frac{8}{5}\right)^2 + \frac{2}{5}\left(\frac{10}{5}\right)^2$$

$$= \frac{2}{5} \cdot \frac{1}{5^2}\left(2^2 + 4^2 + 6^2 + 8^2 + 10^2\right)$$

$$= \frac{2^3}{5^3}\left(1^2 + 2^2 + 3^2 + 4^2 + 5^2\right)$$

$$= \frac{8}{125} \cdot 55$$

$$= \text{etc.}$$

This is an approximation to the area A.

More accurately :

Let $h = \frac{2}{n} \longrightarrow 0$.

$$A = \lim_{h \to 0} \frac{2^3}{n^3} \left(1^2 + 2^2 + 3^2 + \cdots + n^2\right)$$

$$= \lim_{h \to 0} \frac{2^3}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \to \infty} \frac{8}{6} \frac{2n^3 + 3n^2 + n}{n^3}$$

$$= \frac{8}{6} \lim_{n \to \infty} 2 + \frac{3}{n} + \frac{1}{n^2}$$

$$= \frac{8}{6} \cdot 2 = \frac{8}{3}.$$

In summary,

$$\int_0^2 x^2 \, dx = \frac{8}{3}.$$

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