Recall An equation involving a derivative is a differential equation.

e.g.

$$\frac{dy}{dt} = ky \qquad (*)$$

where k is a constant and y is a function of t.

Last week we saw that

$$y = Ae^{kt} \qquad (**)$$

is a solution to $(*)$ for any constant A.

The only solutions to $(*)$ are those of the form $(**)$.

To see this, suppose that

$y = y(t)$ and $z = z(t)$ both satisfy (*). Then

$$\frac{d}{dt}\left(\frac{y}{z}\right) = \frac{z'y - y'z}{z^2}$$

$$= \frac{kzy - kyz}{z^2}$$

$$= 0.$$

So $\frac{y}{z}$ is a constant, say

$\frac{y}{z} = A$, or $y = Az$.

World Population

Aim: what will the world population be in 200 years time?

strategy: start with a simple "model" of population and refine the model if necessary.

Let $y(t)$ = world population at time t, measured in years

Malthusian Law

In 1798 English economist Thomas Malthus suggested that the rate of change of a population (births per year minus deaths per year) is proportional to the size of the population.

Malthusian Law

$$\frac{dy}{dt} = k\,y$$

This means that at time t the world population is

$$y = A e^{kt}$$

for some constants A and k.

Data:

year	world population
1960	3 060 million
(2017	7 400 million)

During the period 1960 - 1980 the world population increased by 2% per year.

Let's start time $t = 0$ in 1960.

$$y(0) = 3.06 \text{ billion} = Ae^{0k} = A$$

$$\boxed{A = 3.06 \text{ billion}}$$

$$y(1) = 1.02\, y(0)$$

$$y(2) = 1.02\, y(1) = 1.02^2\, y(0)$$

$$y(3) = 1.02\, y(2) = 1.02^3\, y(0)$$

$$\boxed{y(t) = (1.02)^t\, y(0)}$$

$$\frac{y(t)}{y(0)} = (1.02)^t = \frac{Ae^{kt}}{A} = e^{kt}$$

$$(1.02)^t = (e^k)^t$$

$$\boxed{e^k = 1.02}$$

$$\ln(e^k) = \ln(1.02)$$

~~$k = \ln(1.02)$~~

$$k \ln(e) = \ln(1.02)$$

$$k = \ln(1.02) \simeq 0.0198$$

$$\boxed{k \simeq 0.0198}$$

$$\boxed{y \simeq 3.06\, e^{0.0198\, t} \quad \text{billion}}$$

This would give todays population as

$$t = 2017 - 1960 = 57$$

$$y = 3.06e^{0.0198 \times 57} = 9.38 \text{ billion.}$$

This is a bit too high!

In fact, in 100 years time our model gives the world population a

$$y = 3.06e^{0.0198 \times 157} = 69.3 \text{ billion}$$

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