MA135 Calculus

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① Integration (8 lectures)

② Techniques of integration (8 lectures)

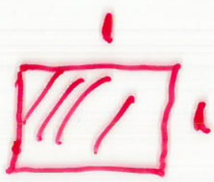
③ Differential equations (8 lectures)

1. Integration

Integration is a theory for calculating areas and volumes that is based on the notion of a limit.

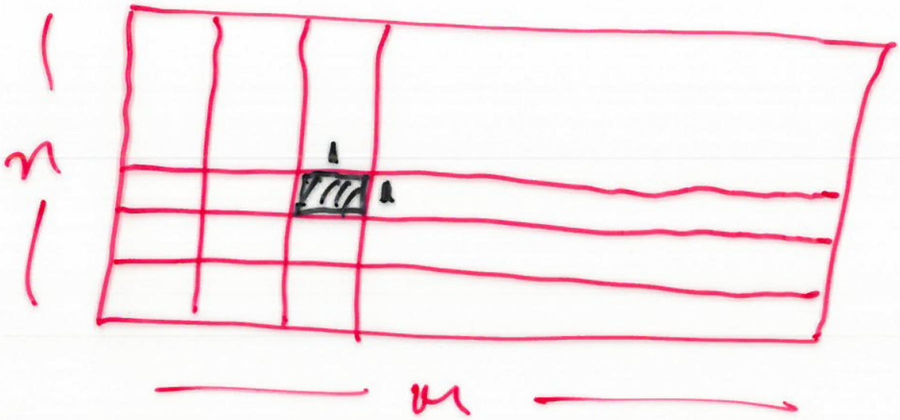
Let's recall some basic details on areas and volumes.

The area of a 1×1 square



is 1

The area of an $m \times n$ rectangle



is mn because the rectangle can be tiled by mn 1×1 squares 

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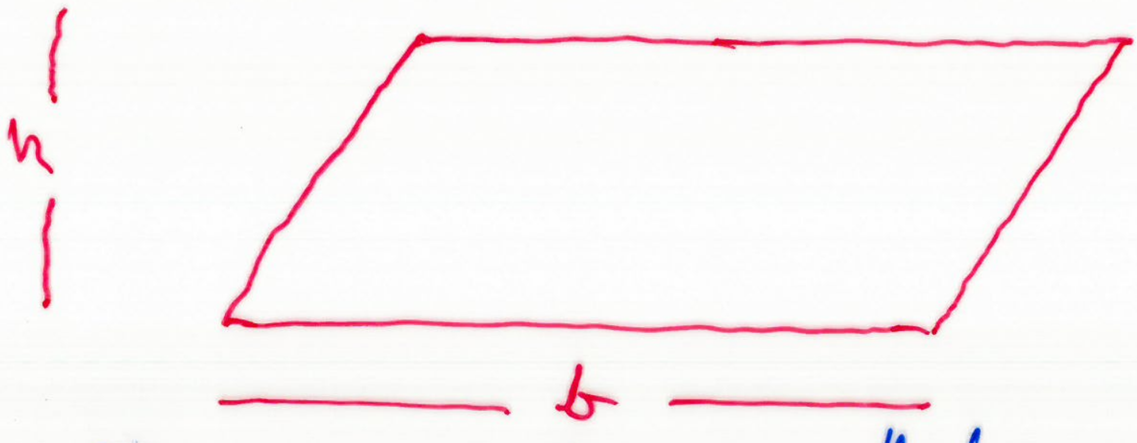
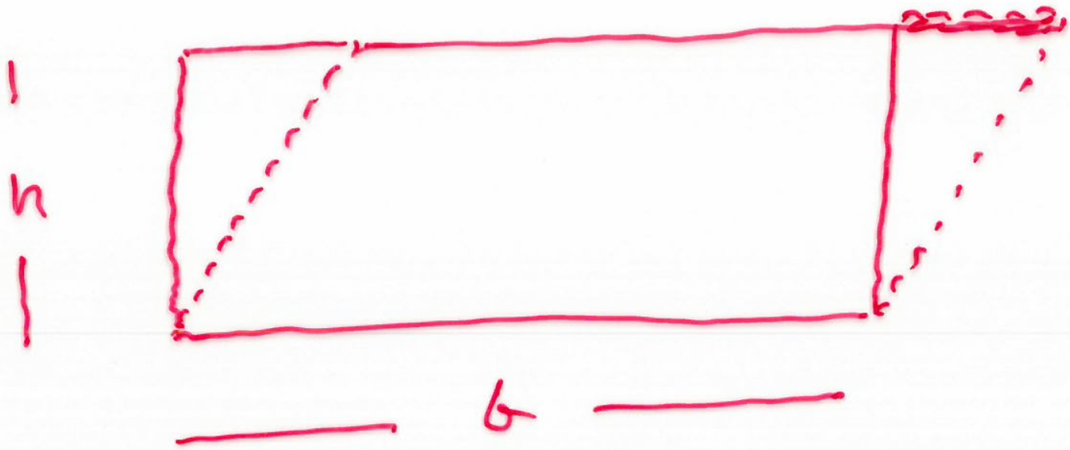
The area of a parallelogram



is bh since the parallelogram can be reconstructed as:

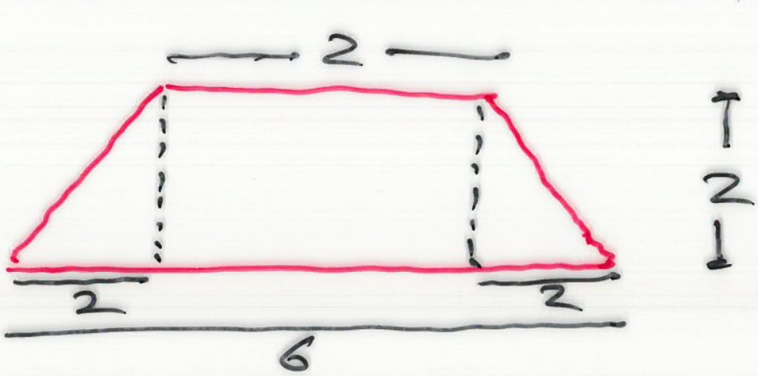


What about the area of a triangle?

area of triangle $= \frac{1}{2} bh = \frac{1}{2}$ base $\times$ perp^{lr} height

$= \frac{1}{2}$ area of parallelogram.

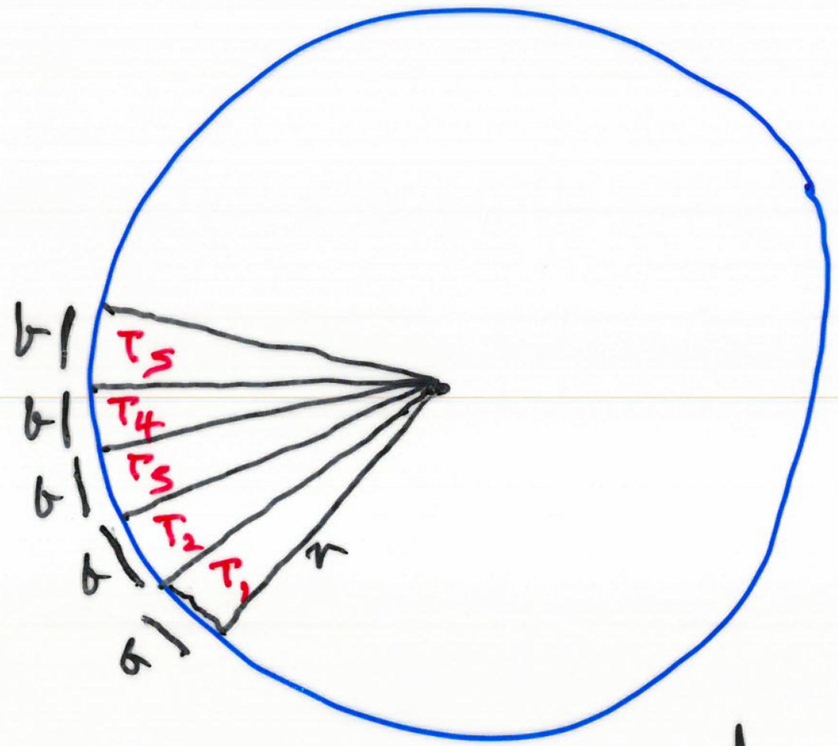
__Example__ Find the area of the region



$$\text{area} = \frac{1}{2}\cdot 2\cdot 2 + 2\times 2 + \frac{1}{2}2\cdot 2$$

$$= 8$$

What about the area of a circle of radius r?



Split the circle into n congruent triangles

$$T_1, T_2, T_3, \cdots, T_n$$

with n large.

Roughly speaking:

Area of circle $\simeq$ area T_1 + area T_2 + ... + area T_n.

$$\simeq \frac{1}{2}br + \frac{1}{2}br + \cdots + \frac{1}{2}br$$

$$= n\,\frac{1}{2}br$$

$$= nb \cdot \frac{r}{2}$$

$$\simeq \text{circumference} \times \frac{r}{2}$$

$$= \frac{r}{2} \cdot 2\pi r$$

$$= \pi r^2 .$$

This approximation for the area of a circle becomes more accurate as we increase the number n of triangles.

In the limit, as $n \to \infty$,

we get

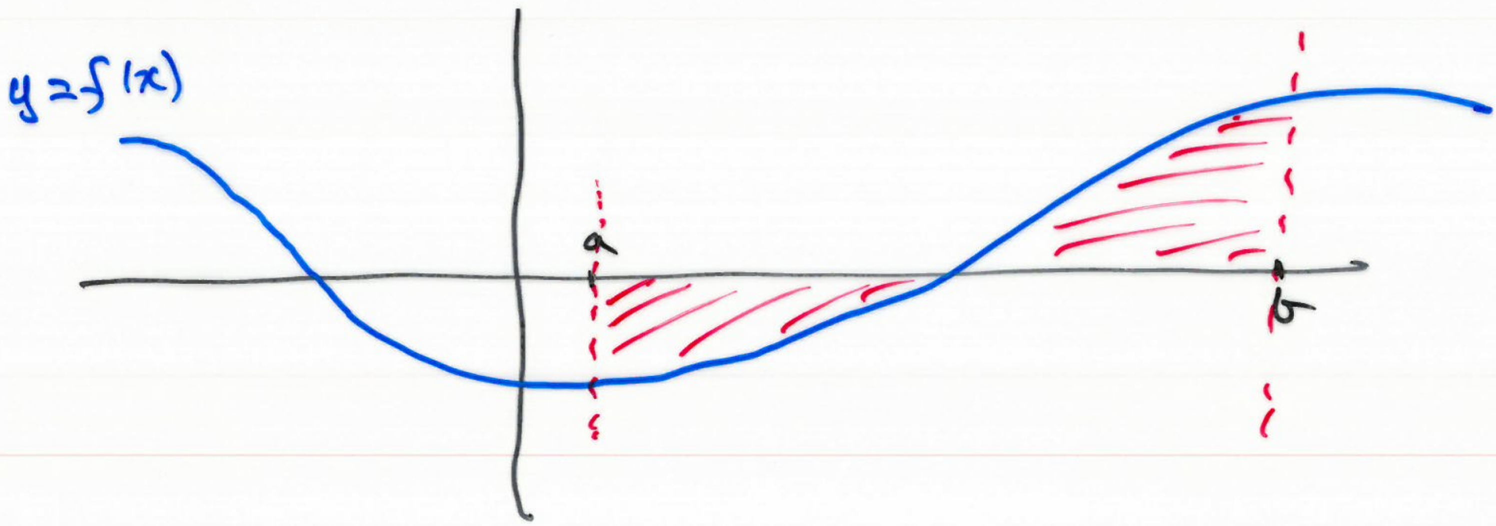
area of circle of radius r	$=$	πr^2

This formula was known to the greeks.

Consider now a function

$$y = f(x)$$

which we picture by a graph:



fix two numbers $a < b$.

We write

$$\int_a^b f(x)\, dx$$

to denote the net area of the region bounded by the curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$. Here "net"

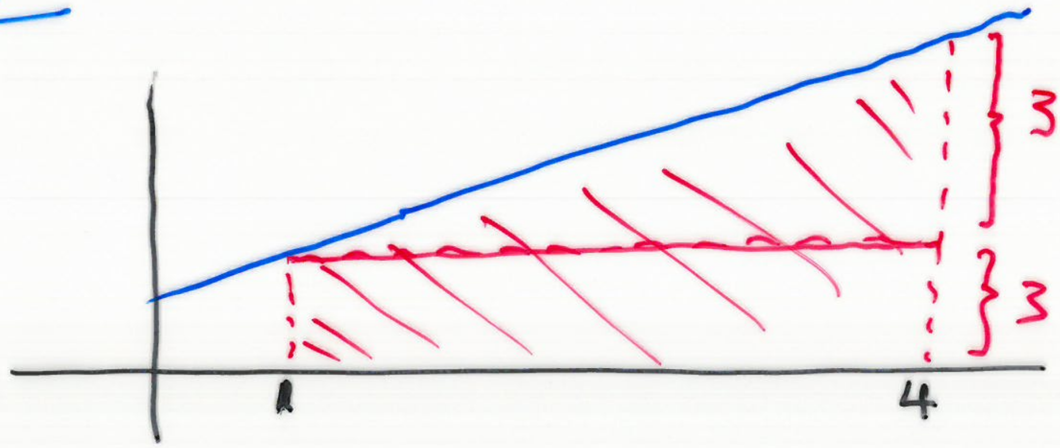
means that areas above the x-axis are positive, and areas below the x-axis are negative.

<u>Example</u> Let $y = x+2$.

Calculate

$$\int_1^4 x+2 \, dx .$$

<u>Solⁿ</u>



$$\int_1^4 x+2 \, dx = \text{area of red region} = 3.3 + \tfrac{1}{2} 3.3 = \frac{27}{2}.$$