Techniques: Simple substitution

Problem Evaluate $I = \int_{-2}^{2} (x-4)^5 \, dx$

Sol^n

Let $u = x - 4$ $\qquad x = -2$ when $u = -6$

$du = dx$ $\qquad x = 2$ when $u = -2$

$$I = \int_{-6}^{-2} u^5 \, du = \left. \frac{u^6}{6} \right|_{-6}^{-2}$$

$$= \frac{(-2)^6}{6} - \frac{(-6)^6}{6}$$

$$= \frac{1}{6}\left(2^6 - 6^6\right)$$

$$= \text{etc.}$$

Problem Evaluate $I = \int_{1}^{2} x\,(x^2+5)^3 \, dx$

Sol^n

Let $u = x^2 + 5$ $\qquad x = 1$ when $u = 6$

$du = 2x\,dx$ $\qquad x = 2$ when $u = 9$

$\frac{1}{2} du = x\,dx$

$$I = \frac{1}{2}\int_6^9 u^3\,du = \frac{u^4}{8}\Bigg|_6^9$$

$$= \frac{9^4}{8} - \frac{6^4}{8} = \frac{3^4}{8}\left(3^4 - 2^4\right).$$

<u>Problem</u> Evaluate

$$I = \int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{2}}} 2x\cos(x^2)\,dx$$

<u>Soln</u>

Let $u = \cancel{2}x^2$ $\qquad x = \sqrt{\frac{\pi}{4}} \Rightarrow u = \frac{\pi}{4}$

$du = 2x\,dx$ $\qquad x = \sqrt{\frac{\pi}{2}} \Rightarrow u = \frac{\pi}{2}$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(u)\,du = \sin(u)\Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)$$

$$= 1 - \frac{1}{\sqrt{2}} \, .$$

Technique 4: Integration by parts

Recall how we differentiate a product.

$$\left(u(x)\,v(x)\right)' = u'(x)\,v(x) + u(x)\,v'(x)$$

so

$$\int (u(x)\,v(x))'\,dx = \int u'(x)\,v(x)\,dx + \int u(x)\,v'(x)\,dx$$

We can rewrite this as

$$\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$$

= write one down

× integral of other

− integral of (one already found × derivative of first)

<u>Problem</u> Find

$$I = \int x \cdot \sin(x)\,dx$$

<u>Sol^n</u>

$$I = x(-\cos(x)) - \int (-\cos(x))\,dx$$

$$= -x\cos(x) + \int \cos(x)\,dx$$

$$= -x\cos(x) + \sin(x) + C.$$