Technique 2: Some standard functions

1) $\int \cos(x)\, dx = \sin(x) + C$

2) $\int \sin(x)\, dx = -\cos(x) + C$

3) $\int e^x\, dx = e^x + C$

4) $\int x^n\, dx = \frac{x^{n+1}}{n+1}, \quad (n \neq -1)$

5) $\int \frac{1}{x}\, dx = ?$

For this, let's recall some basic ideas on logarithms.

$2^2 = 4$ means $\log_2 4 = 2$

$2^3 = 8$ $\quad \log_2 8 = 3$

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\quad \log_2\left(\frac{1}{16}\right) = -4$

$$2^{\log_2 4} = 4$$

$$2^{\log_2 8} = 8$$ $\log_2(-4) =$ not defined

$$2^{\log_2 (\frac{1}{16})} = \frac{1}{16}.$$ $\log_2(0) =$ not defined

<u>Notation</u> we write

$$\ln(x) = \log_e x \quad \text{for } x > 0.$$

"So"

$$\boxed{e^{\ln(x)} = x} \quad \text{for } x > 0.$$

The chain rule for differentiation gives:

$$x = e^{\ln(x)}$$

$$\frac{d}{dx} x = \frac{d}{dx}\left(e^{\ln(x)}\right)$$

$$1 = e^{\ln(x)} \frac{d}{dx} \ln(x)$$

$$1 = x \frac{d}{dx} \ln(x)$$

$$\boxed{\frac{1}{x} = \frac{d}{dx} \ln(x)} \quad \text{for } x > 0$$

Hence

5) $\int \frac{1}{x}\,dx = \ln(x) + c$

$x > 0$.

<u>Problem</u> Determine $\int \frac{1}{x+2}\,dx$).

<u>Sol^n</u> $\frac{d}{dx} \ln(x+2)$

$$= \frac{1}{x+2} \frac{d}{dx}(x+2)$$

$$= \frac{1}{x+2}$$

Hence

$$\int \frac{1}{x+2}\,dx = \ln(x+2).$$

<u>Problem</u> Determine $\int \frac{2x+2}{x^2+2x}\,dx$.

<u>Sol^n</u> $\frac{d}{dx} \ln(x^2+2x)$

$$= \frac{1}{x^2+2x} \frac{d}{dx}(x^2+2x)$$

$$= \frac{1}{x^2+2x}(2x+2)$$

$$= \frac{2x+2}{x^2+2x}$$

Hence

$$\int \frac{2x+2}{x^2+2x}\,dx = \ln(x^2+2x) + C.$$

Problem Find $\displaystyle\int \frac{\cos(x)}{\sin(x)}\,dx$.

Sol^n

$$\int \frac{\cos(x)}{\sin(x)}\,dx = \ln(\sin(x)) + C$$

for $\sin(x) > 0$.

Technique 3 : Simple substitution

To find $\int x \sin(x^2+3)\,dx$ we might just "spot" that

$$\frac{d}{dx} \cos(x^2+3)$$

$$= -\sin(x^2+3) \cdot 2x$$

or $$\frac{d}{dx} -\frac{1}{2}\cos(x^2+3) = x \sin(x^2+3).$$

Hence

$$\int x \sin(x^2+3)\,dx = -\frac{1}{2}\cos(x^2+3) + C.$$

But there is an easier (?) alternative way to do this.

To evaluate

$$I = \int x \sin(x^2+2)\, dx$$

we can substitute

$$u = x^2 + 2$$

$$du = 2x\, dx \quad , \quad \tfrac{1}{2} du = x\, dx$$

Then

$$I = \frac{1}{2}\int \sin(u)\, du$$

$$= -\frac{1}{2}\cos(u) + C$$

$$= -\frac{1}{2}\cos(x^2+2) + C .$$