Lecture 2

1) proposition 1:

$$\int_a^b k\, f(x)\, dx = k \int_a^b f(x)\, dx$$

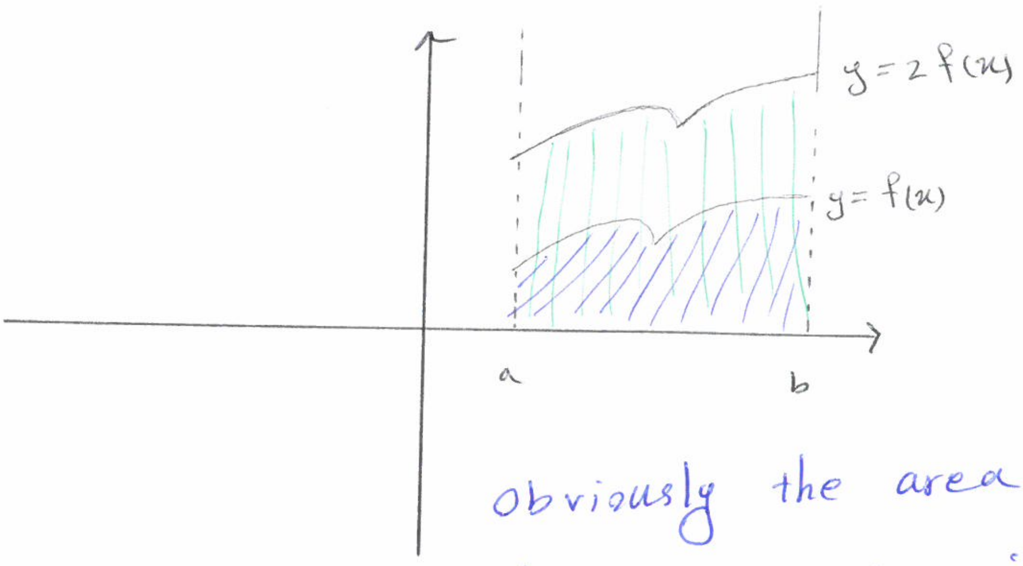
~~we~~ we prove the theorem when $k=2$

the green region

$$\int_a^b 2f(x)\,dx = \text{the area of the region below } \boxed{y = 2f(x)}$$

$$2\int_a^b f(x)\,dx = \boxed{\text{twice}} \text{ the area of the region below } \boxed{y = f(x)}$$

the blue region

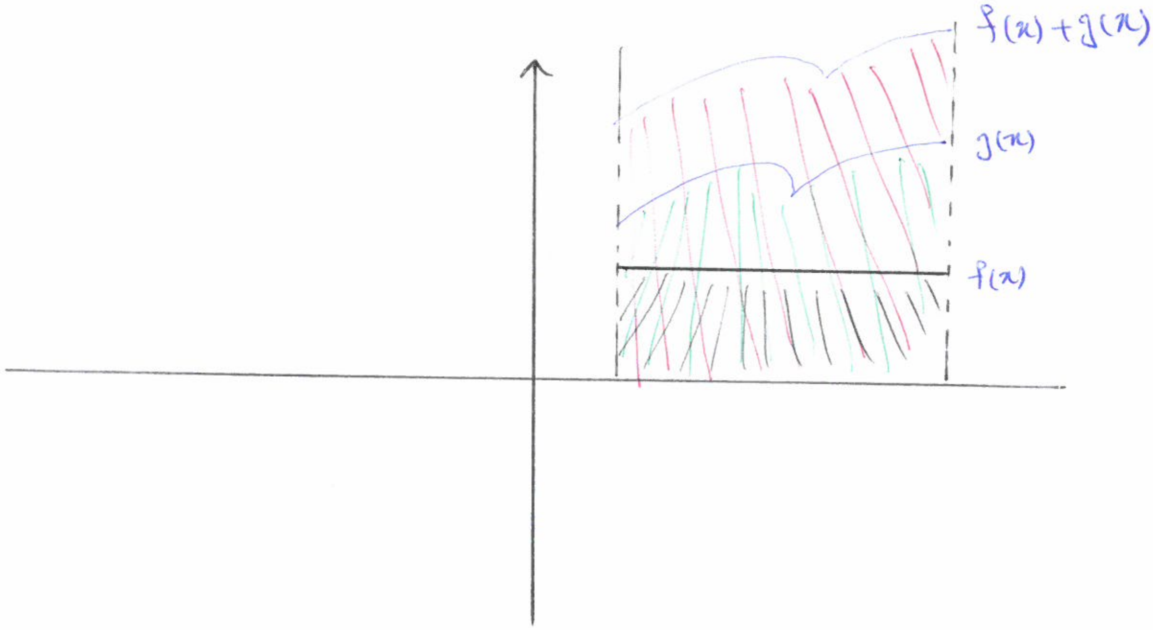


obviously the area of the green region is twice the area of the blue region.

2)

proposition 2:

$$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$



so

the area of the black region + the area of the green region = the area of the red region

ex. Evaluate $I = \int_{-1}^{3} 1 + x + \lfloor x \rfloor^2 \, dx$

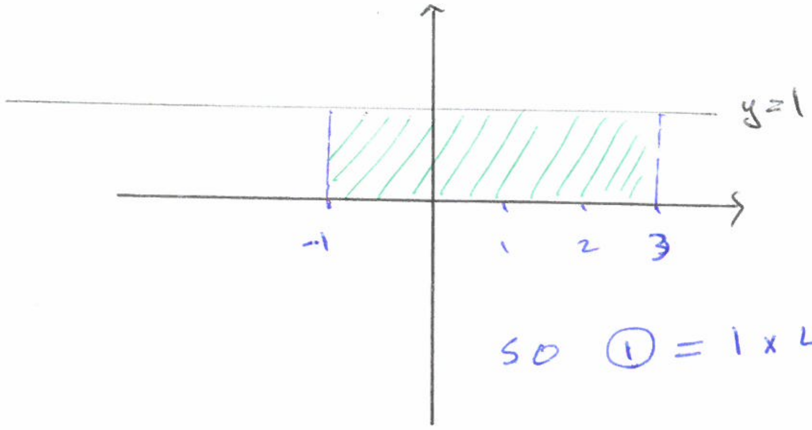
By proposition 2:

$$I = \underbrace{\int_{-1}^{3} 1\,dx}_{1} + \underbrace{\int_{-1}^{3} x\,dx}_{2} + \underbrace{\int_{-1}^{3} \lfloor x \rfloor^2\,dx}_{3}$$

3 = 6
already done!

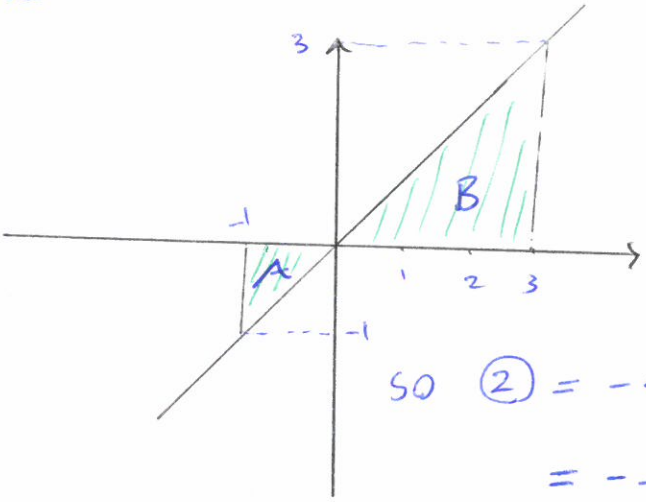
3)

$$① = \int_{-1}^{3} 1\,dx$$



so $① = 1 \times 4 = 4$

$$② = \int_{-1}^{3} x\,dx$$



so $② = -A + B$

$$= -\frac{1}{2} + \frac{3 \times 3}{2}$$

$$= -\frac{1}{2} + \frac{9}{2} = \frac{8}{2} = 4$$

so $I = ① + ② + ③ = 4 + 4 + 6 = \underline{\underline{14}}$

ex. Evaluate $I = \int_{-1}^{3} 1 + x + 2\lfloor x \rfloor^2\,dx$

By proposition 2:

$$I = \int_{-1}^{3} 1\,dx + \int_{-1}^{3} x\,dx + \int_{-1}^{3} 2\lfloor x \rfloor^2\,dx$$