

10.8

Q3 iii) Find $I = \int \frac{2x}{x^2+8} dx$

Soln

$$I = \ln|x^2+8| + C$$

Example $I = \int \frac{x}{x^2+1} dx$

Soln

$$I = \frac{1}{2} \int \frac{2x}{x^2+1} dx$$

$$= \frac{1}{2} \ln|x^2+1| + C$$

Example $I = \int \frac{2 \sin x}{1 + \cos(x)} dx$

Soln

$$I = -2 \int \frac{\sin x}{1 + \cos(x)} dx$$

$$= -2 \ln|1 + \cos(x)| + C$$

8. Factorize

$$x^5 + x^4 + x^3 + x^2 + x + 1$$

as a product of real
linear and quadratic factors.

$$p(x) = (x-1)(x^5 + x^4 + x^3 + x^2 + x + 1) = x^6 - 1$$

$p(x) = 0$ if $x^6 = 1$, i.e. if x is a
6th root of
unity.

Let

$$\omega = e^{\frac{2\pi i}{6}} = e^{\frac{\pi i}{3}}.$$

The 6th roots of unity are

$$\omega^0, \omega, \omega^2, \omega^3, \omega^4, \omega^5.$$

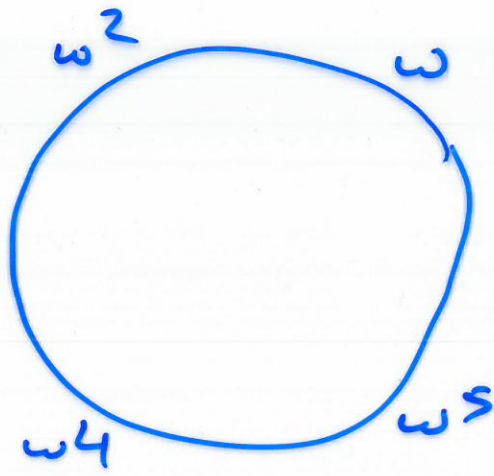
$$p(x) = (x-\omega^0)(x-\omega^1)(x-\omega^2)(x-\omega^3)(x-\omega^4)(x-\omega^5)$$

So

$$x^5 + x^4 + x^3 + x^2 + x + 1 =$$

$$(x-\omega)(x-\omega^2)(x-\omega^3)(x-\omega^4)(x-\omega^5)$$

$$= (x-\omega)(x-\omega^2)(x+1)(x-\omega^4)(x-\omega^5)$$



$$\omega^4 = \overline{\omega^2}$$

$$\omega^5 = \overline{\omega}$$

So

$$x^5 + x^4 + x^2 + x + 1 =$$

$$(x+1) \left[(x-\omega)(x-\overline{\omega}) \right] \left[(x-\omega^2)(x-\overline{\omega^2}) \right]$$

3 (iii)'

$$I = \int \frac{x^2 + x}{x^3 + 3x + 8} dx$$

Soln

CA 100% for 48 correct Qs

40% for 19 correct Qs

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