10.8

1i) $\int (x^2-1)(x+1)\ dx$

$= \int x^3 + x^2 - x - 1 \quad dx$

$= \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - x + C$

2ii) $I = \int \sqrt{2x+1}\ dx$

$u = 2x+1$

$du = 2\,dx \qquad dx = \frac{1}{2}du$

$I = \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$

$= \frac{1}{3}u^{\frac{3}{2}} + C$

$= \frac{1}{3}(2x+1)^{\frac{3}{2}} + C$

10.9

7) Solve

$$\frac{dy}{dt} = 1 + y^2, \quad y(0) = 0 .$$

$$dy = (1+y^2)\,dt$$

$$\boxed{\int \frac{dy}{1+y^2} = \int dt}$$

$$\boxed{\int \frac{dy}{1+y^2} = t + C}$$

substitute $y = \tan\theta$ $\quad t=0, y=0, \theta=0$

$dy = \sec^2\theta\, d\theta$

$$\int \frac{dy}{1+y^2} = \int \frac{\sec^2\theta \, d\theta}{1+\tan^2\theta} = \int d\theta = \theta$$

So $\theta = t + C$

and $0 = 0 + C$ implies $C = 0$.

Hence $\theta = t$ and

$$\boxed{y = \tan t}$$

3 iv) $$I = \int \frac{6x^2 + 4x + 2}{x^3 + x^2 + x + 8}\, dx$$

$$= 2\int \frac{3x^2 + 2x + 1}{x^3 + x^2 + x + 8}\, dx$$

$$= 2\ln|x^3 + x^2 + x + 8| + C.$$