10.9

1) $y = ce^{3x} - e^{2x}$

$$\frac{dy}{dx} = 3ce^{3x} - 2e^{2x}$$

So $\frac{dy}{dx} - 3y$

$= 3ce^{3x} - 2e^{2x} - (3ce^{3x} - 3e^{2x})$

$= e^{2x}$ as required

We want $y(0) = 3$.

So $c = ?$

$3 = y(0) = ce^{3.0} - e^{2.0}$

$= c - 1$

$c = 4$.

2) $y = A e^{kt}$

$$\frac{dy}{dt} = A \frac{d}{dt} e^{kt}$$

$$= A e^{kt} \frac{d}{dt}(kt)$$

$$= A e^{kt} k$$

$$= y k$$

Thus $\frac{dy}{dt} = ky$

$y(t) = A e^{kt}$

$60 = y(0) = A$ $\quad$ $\boxed{A = 60}$

Need also $y(5) = 30$.

What should $k = ?$

$$y = 60\,e^{kt}$$

$$30 = y(5) = 60\,e^{k\cdot 5}$$

$$30 = 60\,e^{5k}$$

$$\frac{1}{2} = e^{5k}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{5k}\right)$$

$$\ln\left(\frac{1}{2}\right) = 5k$$

$$k = \frac{1}{5}\ln\left(\frac{1}{2}\right)$$

5) Solve

$$y^2 \frac{dy}{dt} = t^2, \quad y(0) = 27.$$

$$\int y^2 dy = \int t^2 dt$$

$$\frac{y^3}{3} = \frac{t^3}{3} + C$$

$$y^3 = t^3 + C$$

$$y = (t^3 + C)^{\frac{1}{3}}$$

$$27 = y(0) = (0^3 + C)^{\frac{1}{3}} = C^{\frac{1}{3}}$$

$$C = 27^3 = 3^9$$

6) Solve

$$e^y \frac{dy}{dt} - t - t^3 = 0, \quad y(0) = 1.$$

$$e^y \frac{dy}{dt} = t^3 + t, \quad y(0) = 1$$

$$\int e^y \, dy = \int t + t^3 \, dt$$

$$e^y = \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$\ln(e^y) = \ln\left(\frac{t^2}{2} + \frac{t^4}{4} + c\right)$$

$$y = \ln\left(\frac{t^2}{2} + \frac{t^4}{4} + c\right)$$

$$1 = y(0) = \ln\left(\frac{0^2}{2} + \frac{0^4}{4} + c\right) = \ln(c).$$

$$\boxed{c = e} \qquad y = \ln\left(\frac{t^2}{2} + \frac{t^4}{4} + e\right)$$