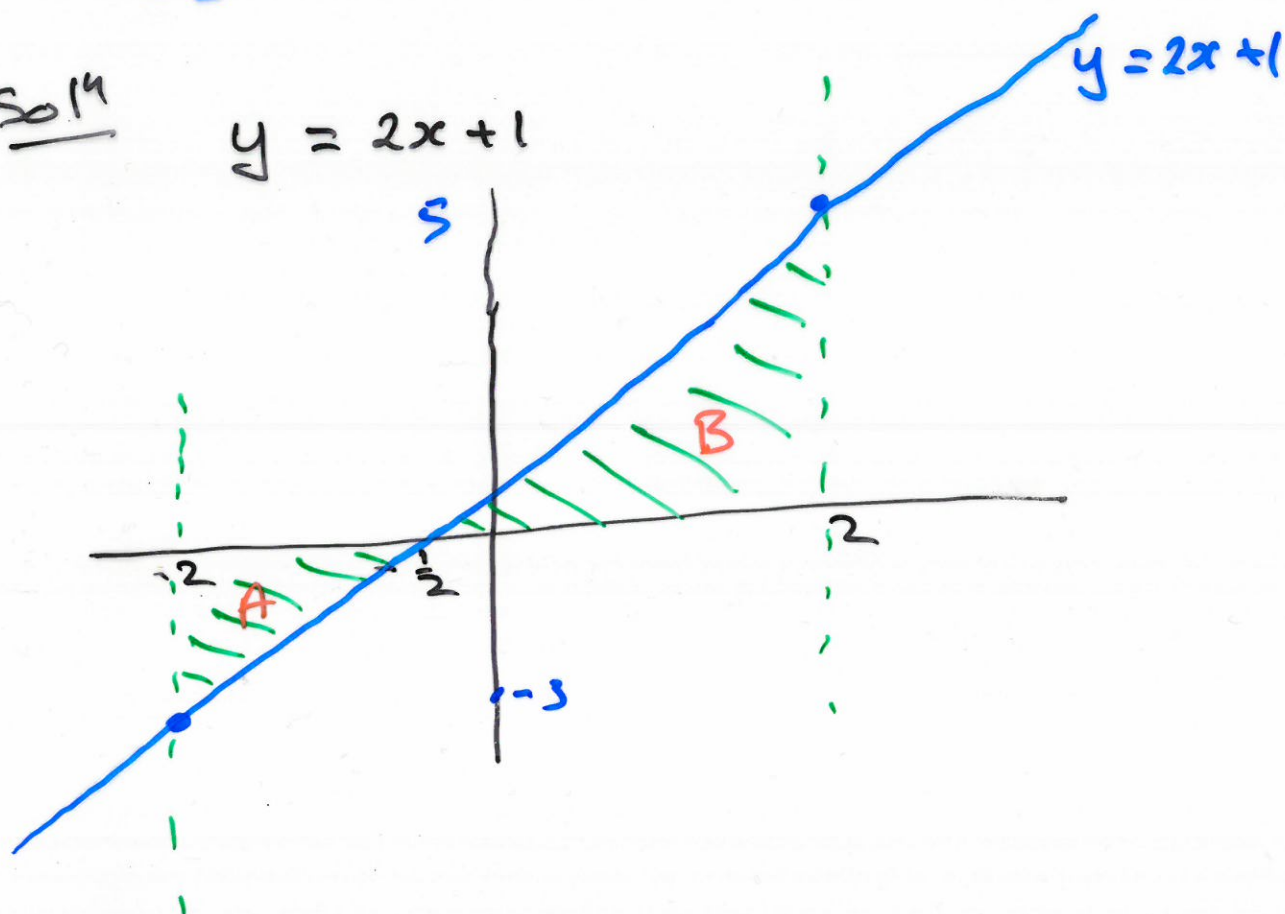


Example Evaluate

$$\int_{-2}^2 2x+1 \, dx$$

Soln



$$\begin{aligned}\int_{-2}^2 2x+1 \, dx &= -A + B \\&= -\frac{1}{2} \cdot \frac{3}{2} \cdot 3 + \frac{1}{2} \cdot \frac{5}{2} \cdot 5 \\&= -\frac{9}{4} + \frac{25}{4} = \frac{16}{4} \\&= 4.\end{aligned}$$

Absolute value function

Define

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

Example

$$|-3| = 3$$

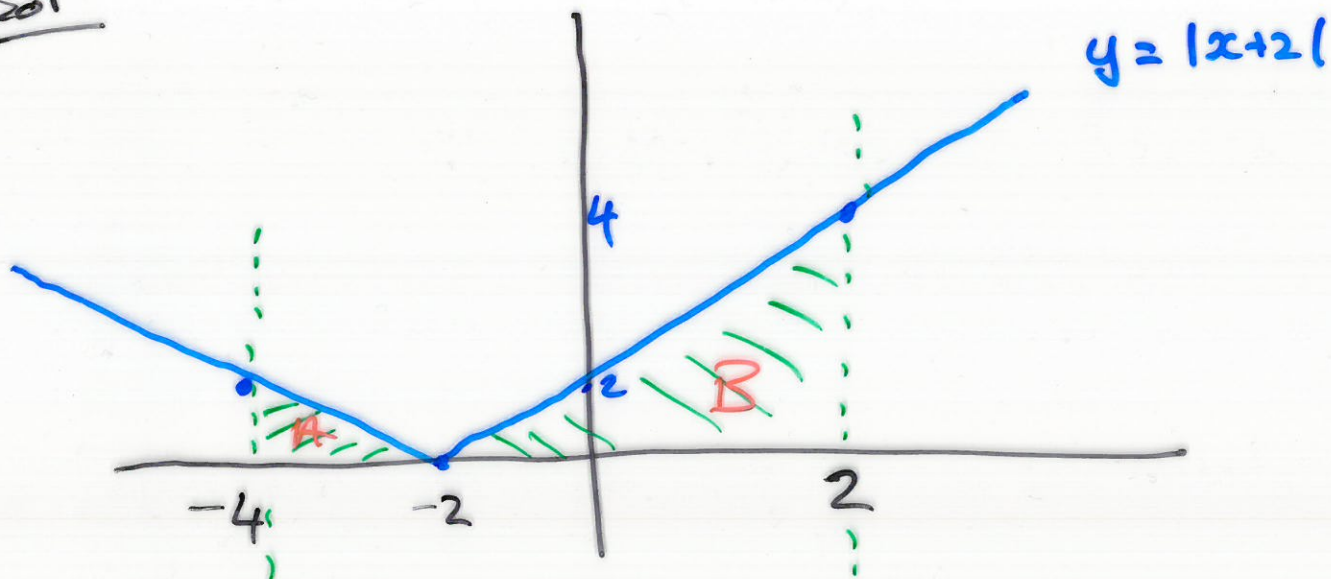
$$|-\frac{1}{2}| = \frac{1}{2}$$

$$|72| = 72$$

Example evaluate

$$\int_{-4}^2 |x+2| dx$$

Soln



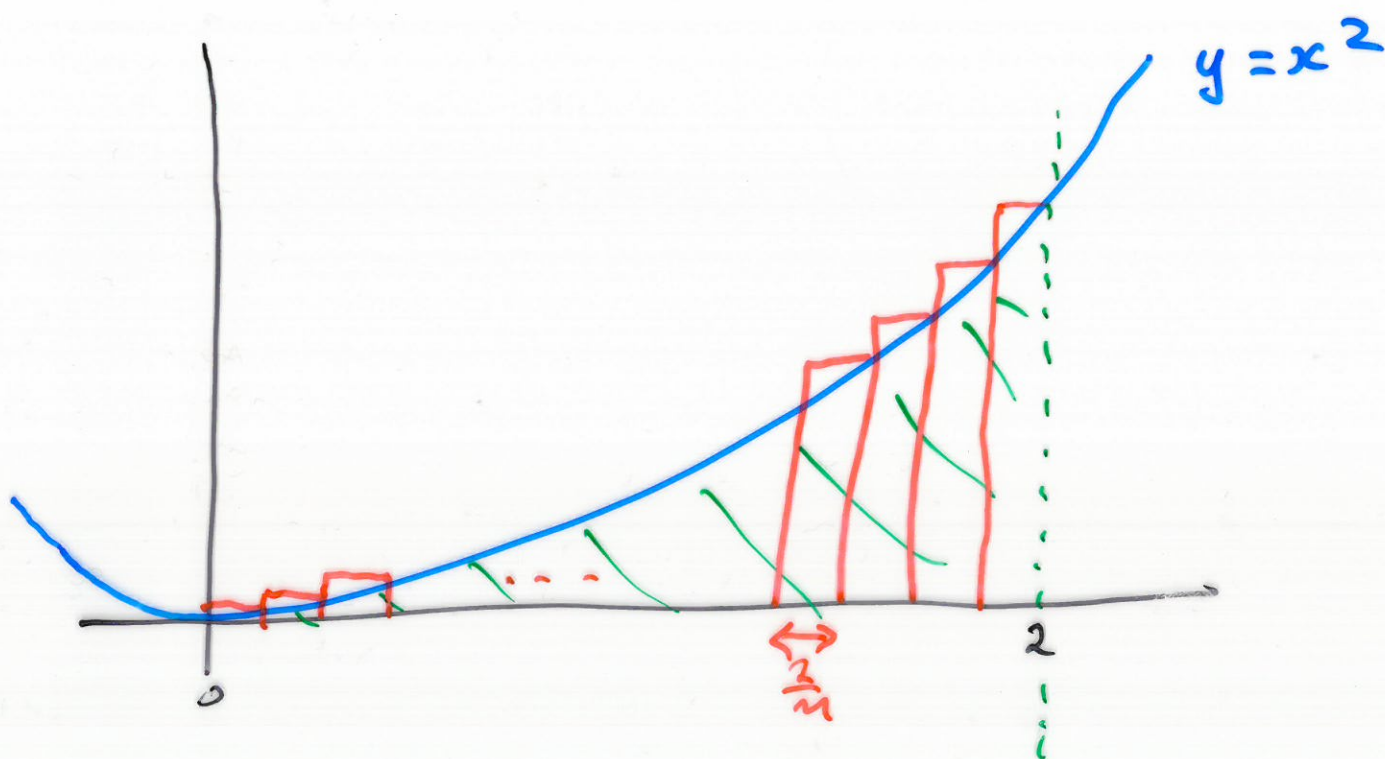
$$\int_{-4}^2 |x+2| dx = A+B$$

$$= \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \cdot 4 \cdot 4$$

$$= 10.$$

How about

$$\int_0^2 x^2 dx$$



Approximate green area as the area of n boxes of equal width.

Let's approximate with $n=5$.

$$A \approx \frac{2}{5} \left(\frac{2}{5} \right)^2 + \frac{2}{5} \left(\frac{4}{5} \right)^2 + \frac{2}{5} \left(\frac{6}{5} \right)^2 + \frac{2}{5} \left(\frac{8}{5} \right)^2 + \frac{2}{5} \left(\frac{10}{5} \right)^2$$

$$= \frac{2}{5} \cdot \frac{1}{5^2} \left\{ 2^2 + 4^2 + 6^2 + 8^2 + 10^2 \right\}$$

$$= \frac{2^3}{5^3} \left\{ 1^2 + 2^2 + 3^2 + 4^2 + 5^2 \right\}$$

$$= \frac{8}{5^3} \cdot 55$$

$$= \text{etc.}$$

More accurately

$$\text{let } h = \frac{2}{n} \rightarrow 0$$

$$A = \lim_{h \rightarrow 0} \frac{2^3}{n^3} \{1^2 + 2^2 + 3^2 + \dots + n^2\}$$

$$= \lim_{h \rightarrow 0} \frac{2^3}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \rightarrow \infty} \frac{8}{6} \frac{2n^3 + 3n^2 + n}{n^3}$$

$$= \frac{8}{6} \lim_{n \rightarrow \infty} 2 + \frac{3}{n} + \frac{1}{n^2}$$

$$= \frac{8}{6} \cdot 2$$

$$= \frac{8}{3} = \int_0^2 x^2 dx$$