Logistic Model: A population $y(t)$ at time $t$ satisfies

$$\frac{dy}{dt} = ky - ly^2 \qquad (*)$$

for $k, l$ constants, $l$ much smaller than $k$. This is a separable differential equation since we can re-write it as

$$\frac{1}{ky-ly^2}\frac{dy}{dt} = 1 \qquad (*)$$

we "integrate both sides" to solve $(*)$

$$\int \frac{1}{ky-ly^2}\, dy = \int dt \qquad (**)$$

For the LHS we need partial fractions.

$$\frac{1}{ky-ly^2} = \frac{1}{y(k-ly)} = \frac{A}{y} + \frac{B}{k-ly}$$

$$\frac{1}{y(k-ly)} = \frac{A(k-ly)+By}{y(k-ly)}$$

$$1 = A(k-ly) + By$$

$y=0:$ $\quad 1 = Ak \qquad \boxed{A = \frac{1}{k}}$

$y = \frac{k}{l}$ $\quad 1 = B\frac{k}{l} \qquad \boxed{B = \frac{l}{k}}$

So (**) becomes

$$\frac{1}{k}\int \frac{1}{y}\,dy + \frac{l}{k}\int \frac{1}{k-ly}\,dy = \int dt$$

$$\frac{1}{k}\ln|y| - \frac{1}{k}\ln|k-ly| = t + c$$

$$\frac{1}{k}\ln\left|\frac{y}{k-ly}\right| = t + c$$

$$\ln\left|\frac{y}{k-ly}\right| = kt + c$$

$$\frac{y}{k-ly} = e^{kt}\underbrace{\boxed{e^{c}}}_{A}$$

$$\frac{y}{k-ly} = A\,e^{kt}.$$

$$y = Ae^{kt}(k-ly)$$

$$y + Ae^{kt}ly = Ake^{kt}$$

$$y(1+Ale^{kt}) = Ake^{kt}$$

$$\boxed{y = \frac{Ake^{kt}}{1+Ale^{kt}}}$$

As $t \to \infty$ we have

$$y \to \frac{Ake^{kt}}{Ale^{kt}} = \frac{k}{l}.$$

Thus the world population will, according to our model, tend to a constant size $\frac{k}{l}$.

- Ecologists have estimated

$$k = 0.029.$$

- when $y = (3.06)10^9$ the population was increasing at 2% per year.

$$\frac{dy}{dt} = ky - ly^2$$

$$\frac{1}{y}\frac{dy}{dt} = k - ly$$

$$0.02 = k - l(3.06)10^9$$

since we know $k$, we can find

$$l = 2.941 \times 10^{-12}.$$

so the world population should tend to

$$\frac{k}{l} = \frac{0.029}{2.941} 10^{12} = 9.86 \text{ billion people}$$