Yesterday

Problem A cup of coffee in a room at 20°C cools from 80°C to 50°C in five minutes. How long will it take to cool to 40°C?

Sol^n We showed that the required time t satisfies

$z(t) = 20$ where:

$$\frac{dz}{dt} = kz \qquad (*)$$

$$z(0) = 60$$
$$z(5) = 30.$$

Any solution to $(*)$ has the form

$$z = Ae^{kt}$$

for some constant A.

$\underline{t = 0}$

$$60 = g(0) = A e^{k \cdot 0} = A e^{0} = A$$

$$\boxed{A = 60}$$

$\underline{t = 5}$

$$30 = g(5) = 60\, e^{k5}$$

$$\boxed{e^{5k} = \frac{1}{2}}$$

For what value of t do we have

$g(t) = 20$?

$$20 = g(t) = 60\, e^{kt}$$

$$\frac{1}{3} = e^{kt}$$

$$\frac{1}{3} = \left(e^{5k}\right)^{\frac{t}{5}}$$

$$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

$$\ln\left(\frac{1}{3}\right) = \ln\left(\left(\frac{1}{2}\right)^{\frac{t}{5}}\right)$$

$$\ln\left(\frac{1}{3}\right) = \frac{t}{5} \ln\left(\frac{1}{2}\right)$$

$$t = 5 \frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{1}{2}\right)} \text{ minutes}$$

World Population

Aim: what will the world population be in 200 years time?

Strategy: start with a simple "model" and modify it if necessary

Let $y(t)$ = world population at time t, measured in years.

Malthusian Law

In 1798 English economist Thomas Malthus suggested that the rate of change of a population (births per year − deaths per year) is proportional to the size of the population.

Malthusian Law:

$$\frac{dy}{dt} = ky$$

This means that at time t the world population is

$$y = A e^{kt}$$

for some constants A, k.

Data:

year	world population
1960	3 060 million

During the period 1960 - 1980 the world population increased by 2% per year.

Let's start time $t=0$ in 1960.

$$y(0) = 3.06 \text{ billion} = Ae^{0k} = A$$

$$\boxed{A = 3.06 \text{ billion}}$$

$$y(1) = 1.02\, y(0)$$

$$y(2) = 1.02\, y(1) = (1.02)^2\, y(0)$$

$$y(3) = 1.02\, y(2) = (1.02)^3\, y(0)$$

$$y(t) = (1.02)^t\, y(0)$$

$$\frac{y(t)}{y(0)} = (1.02)^t = \frac{Ae^{kt}}{A} = e^{kt}$$

so $\boxed{e^k = 1.02}$

$$\ln(e^k) = \ln(1.02)$$

$$k \ln(e) = \ln(1.02)$$

$$k = \ln(1.02) = 0.0198$$

$$\boxed{\text{roughly } k = 0.02}$$

$$y = 3.06\, e^{0.02t}$$

How long does it take for the population to double?

$$y(t) = 2y(0)$$

$$Ae^{0.02t} = 2A$$

$$e^{0.02t} = 2$$

$$\ln(e^{0.02t}) = \ln(2)$$

$$0.02t \ln(e) = \ln(2)$$

$$t = \frac{\ln(2)}{0.02} = 34.66$$

Fact: Over the period 1700 – 1970 the world population doubled every 35 years.

Let's calculate today's population using our model.

$$t = 2016 - 1960 = 56$$

$$y = 3.06\, e^{0.02 \times 56} = 9.38 \text{ billion}$$

This is a bit too high!

In 100 years time our model gives the world population as

$$y = 3.06\, e^{0.02 \times 156} = 69.3 \text{ billion}$$

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