

## Techniques: Integration by parts

$$\int u(x) u'(x) dx = u(x) v(x) - \int v(x) u'(x) dx$$

Integral of  
Product = write one down  
× integral of other  
- integral of (one already found  
× derivative of first)

Problem Find

$$I = \int x \sec^2 x \, dx$$

Sol<sup>n</sup> (Recall  $\frac{d}{dx} \tan x = \sec^2 x$ )

$$I = x \tan x - \int \tan x \, dx$$

$$= x \tan x + \int \frac{-\sin x}{\cos x} \, dx$$

$$= x \tan x + \ln(\cos x) + C.$$

Problem Evaluate

$$I = \int \ln(x) \, dx$$

Sol<sup>n</sup>

$$I = \int 1 \cdot \ln(x) \, dx$$

$$= x \ln(x) - \int x \frac{1}{x} \, dx$$

$$= x \ln(x) - \int 1 \, dx$$

$$= x \ln(x) - x + C.$$

Problem Evaluate

$$I = \int x e^x \, dx.$$

Sol<sup>n</sup>

$$I = x e^x - \int e^x \, dx$$

$$= x e^x - e^x + C.$$

Problem Evaluate

$$I = \int x \ln(x) \, dx$$

Soln

$$I = \frac{x^2}{2} \ln(x) - \int \frac{x^2}{2} \frac{1}{x} \, dx$$

$$= \frac{x^2}{2} \ln(x) - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} \ln(x) - \frac{x^2}{4} + C.$$



Problem Prove that

$$\int \sin^n(x) dx = -\frac{1}{n} \cos(x) \sin^{n-1}(x) + \frac{n-1}{n} \int \sin^{n-2} x dx$$

for integers  $n \geq 2$ .

Sol<sup>n</sup>

$$\begin{aligned} I &= \int \underbrace{\sin(x)}_{u'} \cdot \underbrace{\sin^{n-1}(x)}_u dx \\ &= -\cos(x) \sin^{n-1}(x) + \int \cos(x) (n-1) (\sin x)^{n-2} \cos(x) dx \\ &= -\cos(x) \sin^{n-1}(x) + (n-1) \int \cos^2(x) \sin^{n-2}(x) dx \\ &= -\cos(x) \sin^{n-1}(x) + (n-1) \int (1 - \sin^2 x) \sin^{n-2}(x) dx \\ &= -\cos(x) \sin^{n-1}(x) + (n-1) \int \sin^{n-2}(x) - \sin^n(x) dx \end{aligned}$$

So

$$I = -\cos(x) \sin^{n-1}(x) + (n-1) \int \sin^{n-2}(x) dx$$

$$- (n-1) I$$

$$nI = -\cos(x) \sin^{n-1}(x) + (n-1) \int \sin^{n-2}(x) dx$$

So

$$I = -\frac{1}{n} \cos(x) \sin^{n-1}(x) + \frac{n-1}{n} \int \sin^{n-2} dx.$$

Name	id	BA	CS
Aleksandra Dabrowske	MO.14104202	✓	
Naigonele Sikwe	14 10 41 89	✓	
Marc Blain	15349941		✓
Dermid Berne	15331436		✓
Steven Noonan	15517507		✓
Thomas Nutley	15532187	✓	
Karen Butler	15356656	✓	
Amber Byrne	15337266	✓	
Chika Onyia	15736825		✓
Alice Hager	15343221	✓	
Aileen Doherty	14331451	✓	
JACEK SZUMSKI	14102325		✓
Karin Lusack	14102730		✓
Domas Petrauskas	14102632		✓
Jack Foley	15478258		✓
Eoghan Hennelly	15480908		✓
James Idehir	13318731		✓
Conor Redington	15332311		✓
Alan Fahy	14104204		✓
Conor Buckley	15534413		✓
Sarah Davies	15321951	✓	
Eamonn Hannon	15310091		✓
Michelle Barnett	15356686	✓	
Ciara McEllistorm	15449178		✓
Emma Moran	15478872	✓	
Aisling Ward	15309706	✓	
Barry Murphy	15747621		✓
Sinead McAlinchey	15314576	✓	
Philip Mae Loidh	15387451		✓
Shanon Smith	15429028	✓	
Ashleigh Hyland	15525283	✓	
Aine McPhillips	15361026	✓	
Declan Wilson	15718721	✓	



BA

CS

Thomas Biggins 15416652 ✓

Jesse Wood 15100665 ✓

Mark Harrington 15488702

Michelle Doherty 15372296 ✓

Stephen O'Brien 15455232 ✓

Chloe Kenny 15354926 ✓

Joanne Eko 14506467 ✓

Daniel Regan 15511417

Amar Mulla 15910831 ✓