MA135 Calculus

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① Integration (8 lectures)

② Techniques of integration (8 lectures)

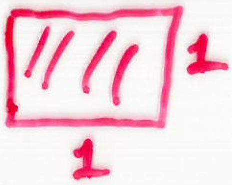
③ Differential Equations (8 lectures)

1. Integration

> Integration is a theory for calculating areas and volumes that is based on the notion of a limit.

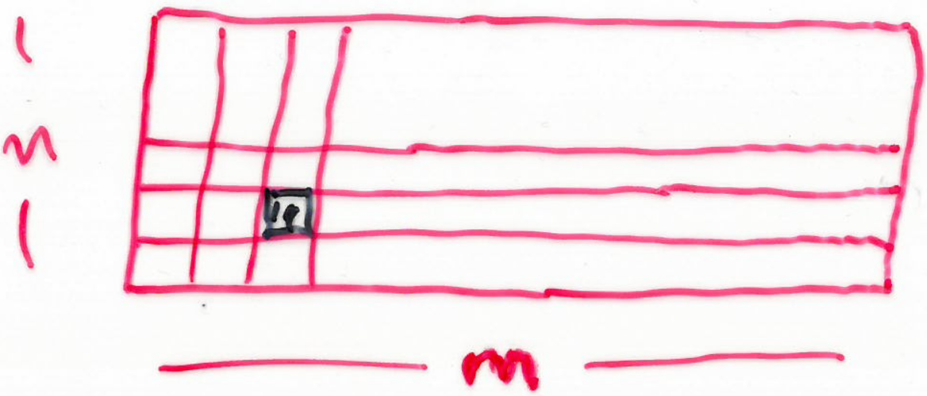
Let's first recall details on areas / volumes.

The area of a 1×1 square



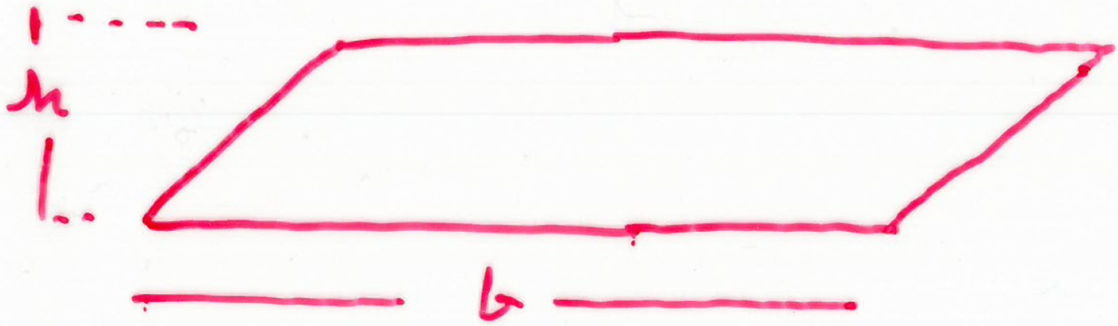
is 1.

The area of an $n \times m$ rectangle



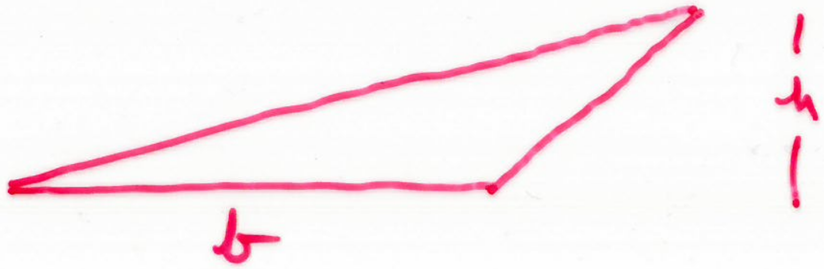
is mn because the rectangle can be tiled by mn squares of type: 

The area of a parallelogram



is bh since because the parallel^{gm} can be reconfigured as:

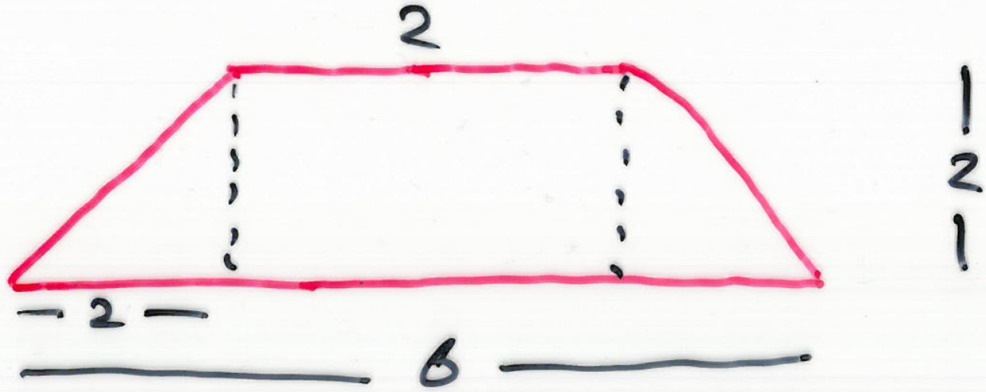
So what about the area of a triangle?



area of triangle $= \frac{1}{2} b h = \frac{1}{2}$ base $\times$ perp. height

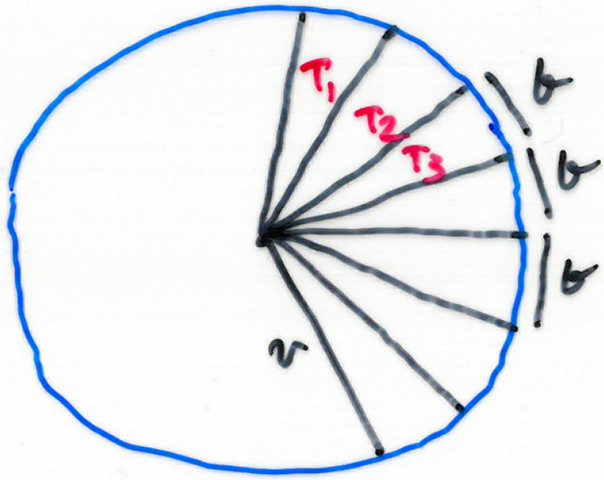
$= \frac{1}{2}$ area of a parallel^{gm}.

<u>Example</u> Find the area of the region



area $= \frac{1}{2}.2.2 + 2.2 + \frac{1}{2}.2.2 = 8$

What about the area of a circle of radius r.



Split the circle into n congruent triangles $T_1, T_2, \ldots, T_n$, n large.

Roughly speaking:

$$\boxed{\text{area of circle}} \simeq \text{area } T_1 + \text{area } T_2 + \cdots + \text{area } T_n$$

$$\simeq \tfrac{1}{2} b r + \tfrac{1}{2} b r + \cdots + \tfrac{1}{2} b r$$

$$= \left(\frac{nb}{2}\right) r$$

$$\simeq \frac{r}{2} \times \text{circumference}$$

$$= \frac{r}{2} 2\pi r = \boxed{\pi r^2}$$

The approximation for the area of a circle becomes more accurate as we increase the number n of triangles.

In the limit as $n \to \infty$ we get

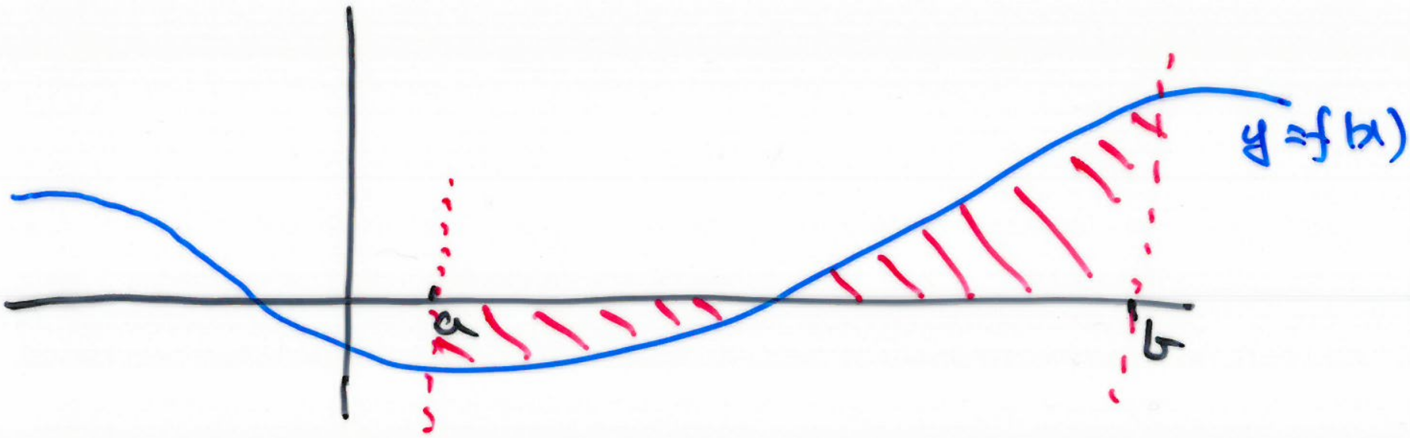
$$\boxed{\text{area of circle of radius } r \;=\; \pi r^2}$$

This use of limits to find an area was known to the ancient Greeks.

Consider now a function

$$y = f(x)$$

which we picture by a graph:



Fix two numbers $a < b$.

We write

$$\int_a^b f(x)\,dx$$

to denote the net area of the region bounded by the curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$. Here "net"

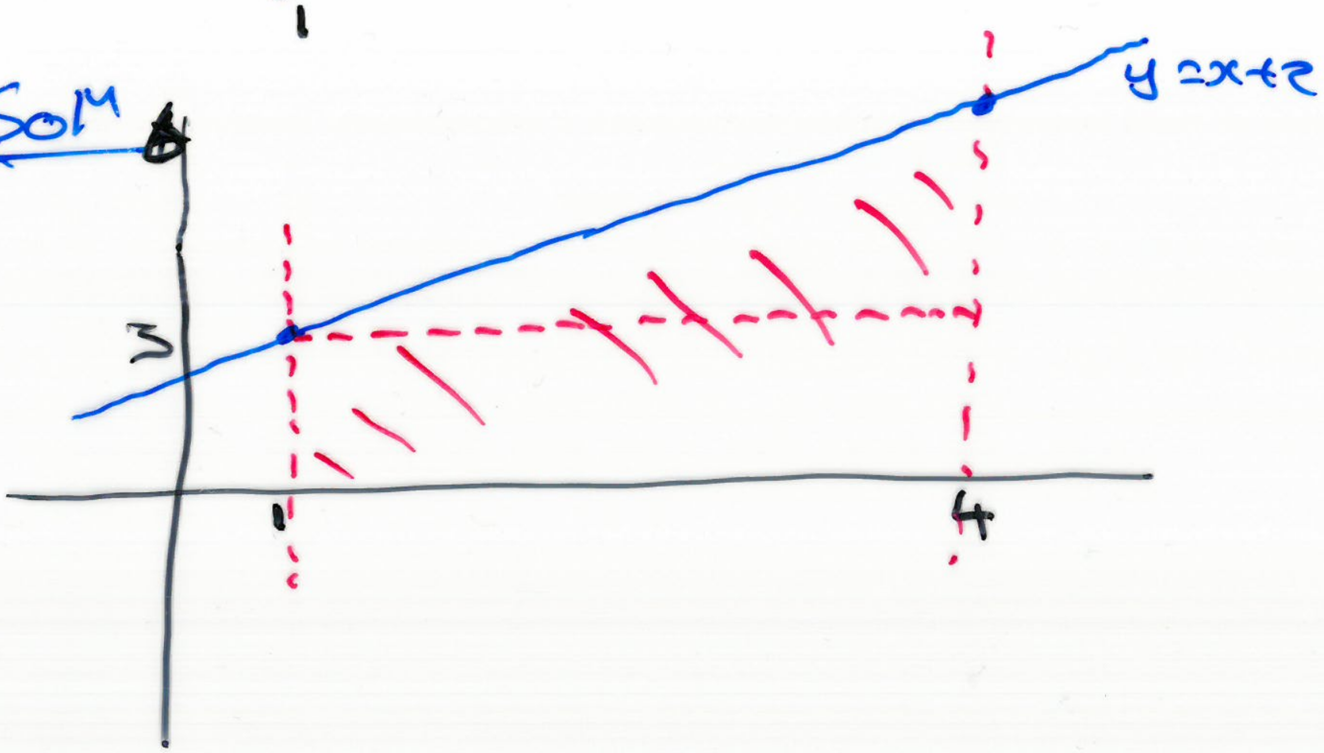
means that areas above the x-axis are positive, and areas below x-axis are negative.

<u>Example</u> Let $y = x+2$.

Calculate

$$\int_1^4 x+2 \ dx .$$

Sol^n



$$\int_1^4 x+2 \ dx = \begin{array}{l}\text{area of}\\ \text{red}\\ \text{region}\end{array} = 3.3 + \tfrac{1}{2}3.3 = \frac{27}{2}.$$