

Summary of logic

- Basic logical connectives

\wedge \vee \neg

- Also important is

\Rightarrow

It is possible to express the function
 \Rightarrow in terms of \wedge, \vee, \neg .

- A truth function is called a tautology if its value is always true no matter what the input is.

Another useful connective

we write $P \Leftrightarrow Q$ as short-hand
for the function $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

A logical argument can be expressed as a truth function (see below).

Definition

An argument is logically valid if the corresponding truth function is a tautology.

Problem is the following logically valid.

If Murphy is a Communist,
Murphy is an atheist. Murphy
is an atheist. Hence Murphy is
a Communist.

$$((P \Rightarrow Q) \wedge Q) \Rightarrow P$$

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \wedge Q$	$((P \Rightarrow Q) \wedge Q) \Rightarrow P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Since this is not a tautology
the argument is not logically
valid.

Problem is the following logically valid?

If capital investment remains constant,
the government spending will
increase. If government spending
will not increase, taxes can be
reduced. If taxes can be reduced
and capital investment remains
constant, then unemployment will
not result. Hence government
spending will increase.

$$[(P \Rightarrow Q) \wedge (\neg Q \Rightarrow R) \wedge ((R \wedge P) \Rightarrow \neg S)] \Rightarrow Q$$

we'll analyse this tomorrow.