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problem: Determine the truth table for the logical

expression:  $(\neg P) \vee (P \wedge Q)$

$\neg$  represents "not p"

$\vee$  "or"

$\wedge$  "and"

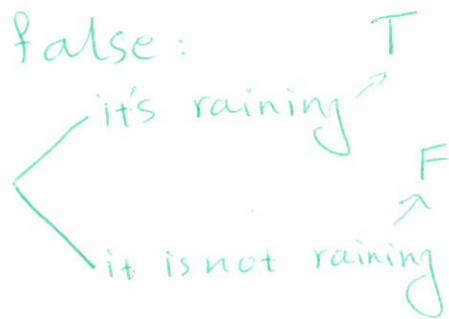
So  $(\neg P) \vee (P \wedge Q)$  equals "(not p) or (p and Q)"

| P | Q | $\neg P$ | $P \wedge Q$ | $(\neg P) \vee (P \wedge Q)$ |
|---|---|----------|--------------|------------------------------|
| T | T | F        | T            | T                            |
| T | F | F        | F            | F                            |
| F | T | T        | F            | <del>T</del>                 |
| F | F | T        | F            | T                            |

remark: Each statement has a truth value,

it could be either true or false:

for example:  $p = \text{it is raining}$



so the number of possibilities

with one statement is  $2^1$

and for 2 statements is  $2^2$

2]

in general for  $n$  statements, we have  $2^n$  possibilities.

problem: For each of the Boolean functions

(i)  $f(x, y) = x \cdot y \pmod{2}$

(ii)  $f(x, y) = x + y \pmod{2}$

complete the truth table.

(i)

| $x$                | $y$ | $f(x, y) = x \cdot y$ |
|--------------------|-----|-----------------------|
| $T \leftarrow (1)$ | 1   | 1                     |
| 1                  | 0   | 0                     |
| $F \leftarrow (0)$ | 1   | 0                     |
| 0                  | 0   | 0                     |

(ii)

| $x$ | $y$ | $f(x, y) = x + y$ |
|-----|-----|-------------------|
| 1   | 1   | 0                 |
| 1   | 0   | 1                 |
| 0   | 1   | 1                 |
| 0   | 0   | 0                 |

$\rightarrow 1+1=2 \equiv 0$

3)

Problem: Determine a function  $f(x, y) \bmod 2$

with values

| $x$ | $y$ | $f(x, y)$ |
|-----|-----|-----------|
| 1   | 1   | 1         |
| 1   | 0   | 0         |
| 0   | 1   | 1         |
| 0   | 0   | 1         |

$$f(x, y) = 1 + x + xy \bmod 2$$

conditional statement

$$P \Rightarrow Q$$

we read it as "P implies Q" or "if P then Q"

for example consider this statement

if it rains, I will stay home.

P Q

| P | Q | $P \Rightarrow Q$ |
|---|---|-------------------|
|---|---|-------------------|

T

T

T

→ it means it rained and I stayed at home

T

F

F

↪ it rained and I didn't stay at home so it is false because I was supposed to stay home when it rains.

F

T

T

F

F

T

it didn't read so

~~it didn't~~ I can stay home or go out both are true

4)

Notation: As you see the two statements

$(\neg P) \vee (P \wedge Q)$  and  $P \Rightarrow Q$  have the same truth table.

ex: which of the following propositions are true?

1)  $\underbrace{2+2=4}_T$  implies Dublin is the capital of Ireland  
 $\Rightarrow T$  so it is T

2)  $\underbrace{2+2=4}_T$  implies Cardiff is the capital of Ireland  
 $\Rightarrow F$  so it is F

3)  $\underbrace{2+2=5}_F$  implies Dublin is the capital of Ireland  
 $\Rightarrow T$  so it is T

4)  $\underbrace{2+2=5}_F$  implies Graham Ellis is the Pope  
 $\Rightarrow F$  so it is F

5)

Def: A logical expression is a tautology if it is always true.

example:  $P \vee \neg P$  is a tautology

| P | $\neg P$ | $P \vee \neg P$ |
|---|----------|-----------------|
| T | F        | T               |
| F | T        | T               |

Def: A logical expression is a contradiction if it is always false

example:  $P \wedge \neg P$  is a contradiction

| P | $\neg P$ | $P \wedge \neg P$ |
|---|----------|-------------------|
| T | F        | F                 |
| F | T        | F                 |

problem: Show that  $(P \wedge \neg P) \Rightarrow Q$  is a

tautology:

| P | Q | $\neg P$ | $P \wedge \neg P$ | $(P \wedge \neg P) \Rightarrow Q$ |
|---|---|----------|-------------------|-----------------------------------|
| T | T | F        | F                 | T                                 |
| T | F | F        | F                 | T                                 |
| F | T | T        | F                 | T                                 |
| F | F | T        | F                 | T                                 |