1)

problem: Determine the truth table for the logical expression: $(\neg P) \vee (P \wedge Q)$

$\neg$ represents "not p"

$\vee$ " "or"

$\wedge$ " "and"

So $(\neg P) \vee (P \wedge Q)$ equals "(not p) or (p and Q)

P	Q	$\neg P$	$P \wedge Q$	$(\neg P) \vee (P \wedge Q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

remark: Each statment has a truth value, it could be either true or false:

for example: p = it is raining $\begin{cases} \text{it's raining} \rightarrow T \\ \text{it is not raining} \rightarrow F \end{cases}$

so the number of possibilites with one statement is 2^1

and for 2 statements is 2^2

2)

in general for n statements, we have

2^n possibilities.

problem: For each of the Boolean functions

(i) $f(x,y) = x \cdot y \mod 2$

(ii) $f(x,y) = x + y \mod 2$

complete the truth table.

(i)

x	y	$f(x,y) = x \cdot y$
T ← 1	1	1
1	0	0
F ← 0	1	0
0	0	0

(ii)

x	y	$f(x,y) = x + y$
1	1	0
1	0	1
0	1	1
0	0	0

$\rightarrow 1 + 1 = 2 \overset{0}{\equiv} 0$

3)

Problem: Determine a function $f(x,y) \bmod 2$

with values

x	y	$f(x,y)$
1	1	1
1	0	0
0	1	1
0	0	1

$f(x,y) = 1 + x + xy \bmod 2$

conditional statement

$P \Rightarrow Q$

we read it as "p implies Q" or "if p then Q"

for example consider this statement

if $\underbrace{\text{it rains}}_{P}$, $\underbrace{\text{I will stay home.}}_{Q}$

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

→ it means it rained and I stayed at home

→ it rained and I didn't stay at home so it is false because I was supposed to stay home when it rains.

it didn't read so ~~it didn't~~ I can stay home or go out both are true

4)

Notation: As you see the two statements $(\neg P) \vee (P \wedge Q)$ and $P \Rightarrow Q$ have the same truth table.

ex: which of the following propositions are true;

1) $\underline{2+2=4}$ implies $\underline{\text{Dublin is the capital or Ireland}}$

T $\Rightarrow$ T so it is T

2) $\underline{2+2=4}$ implies $\underline{\text{cardiff is the capital of Ireland}}$

T $\Rightarrow$ F so it is F

3) $\underline{2+2=5}$ implies $\underline{\text{Dublin is the capital of Ireland}}$

F $\Rightarrow$ T so it is T

4) $\underline{2+2=5}$ implies $\underline{\text{Graham Ellis is the Pope}}$

F $\Rightarrow$ F so it is T

5)

Def: A logical expression is a tautology if it is always <u>true</u>.

example: $P \vee \neg P$ is a tautology

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

Def: A logical expression is a contradiction if it is always <u>false</u>

example: $P \wedge \neg P$ is a contradiction

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

problem: Show that $(P \wedge \neg P) \Rightarrow Q$ is a tautology:

P	Q	$\neg P$	$P \wedge \neg P$	$(P \wedge \neg P) \Rightarrow Q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T