5) Find z such that

$$z^2 = -15 - 8i$$

We need $z = x + iy$ such that

$$(x+iy)(x+iy) = -15 - 8i$$

$$\left.\begin{aligned} x^2 - y^2 &= -15 \\ 2xy &= -8 \end{aligned}\right\} \quad (*)$$

~~x~~ $y = -\dfrac{4}{x}$

(*)

$$x^2 - \frac{16}{x^2} = -15$$

$$x^4 - 16 = -15x^2$$

$$\boxed{x^4 + 15x^2 - 16 = 0} \quad (*)$$

Let $w = x^2$, so $w \geq 0$.

* $w^2 + 15w - 16 = 0$

$(w + 16)(w - 1) = 0$

So $\omega = 1$, $x^2 = 1$, $x = \pm 1$

Thus

$z = 1 - 4i$

and

$z = -1 + 4i$

are the two square roots of

$-15 - 8i$.

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6) The cube roots of 1 are

$\omega^0 = e^{0i} = 1$ $\qquad 1^3 = 1$

$\omega = e^{\frac{2\pi}{3}i}$ $\qquad \omega^3 = 1$

$\omega^2 = e^{\frac{4\pi}{3}i}$ $\qquad (\omega^2)^3 = 1$

$x^3 - 1 = (x-1)(x-\omega)(x-\omega^2)$

7) De Moivre :

$$(\cos\theta + i\sin\theta)^3 = (\cos 3\theta + i\sin 3\theta)$$

$$(\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$$

$$= \cos 3\theta + i\sin 3\theta$$

$$(\cos\theta + i\sin\theta)(\cos^2\theta - \sin^2\theta + i\,2\sin\theta\cos\theta)$$

$$= \cos 3\theta + i\sin 3\theta$$

$$\underbrace{\cos^3\theta - \cos\theta\sin^2\theta - \sin^2\theta\cos\theta} + i(\ldots\ldots)$$

$$= \underbrace{\cos 3\theta} + i\sin 3\theta$$

$$\cos 3\theta = \cos^3\theta - \cos\theta\sin^2\theta - \sin^2\theta\cos\theta$$

$$\cos 3\theta = \cos^3\theta - \cos\theta(1-\cos^2\theta) - (1-\cos^2\theta)\cos\theta$$

$$\boxed{\cos 3\theta = 3\cos^3\theta - 2\cos\theta}$$

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