

10.11

$$1) \quad w = 5 + 5i \quad z = 3 - 4i$$

$$\frac{w}{z} = \frac{(5 + 5i)}{(3 - 4i)} \cdot \frac{(3 + 4i)}{(3 + 4i)}$$

$$= \frac{15 - 20 + 15i + 20i}{9 + 16}$$

$$= \frac{-5 + 35i}{25} = -\frac{1}{5} + \frac{7}{5}i$$

$$4) \quad z = \frac{5 + 5i}{3 - 4i} + \frac{20}{4 + 3i}$$

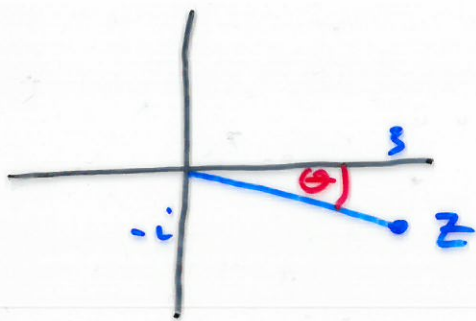
$$= -\frac{1}{5} + \frac{7}{5}i + \frac{(20)}{(4 + 3i)} \cdot \frac{(4 - 3i)}{(4 - 3i)}$$

$$= -\frac{1}{5} + \frac{7}{5}i + \frac{80 - 60i}{25}$$

$$= -\frac{1}{5} + \frac{7}{5}i + \frac{16 - 12i}{5}$$

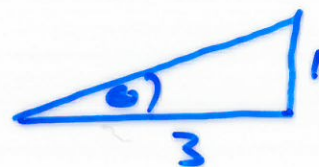
$$\boxed{z = 3 - i}$$

$$|z| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$



$$\text{Arg}(z) = -\theta$$

$$\text{where } \tan \theta = \frac{1}{3}$$



$$3) \quad \text{Arg}(v) = 15^\circ$$

$$|v| = 2$$

$$\text{Arg}(w) = 45^\circ$$

$$|w| = 4$$

$$z = \frac{v^7}{w^3}$$

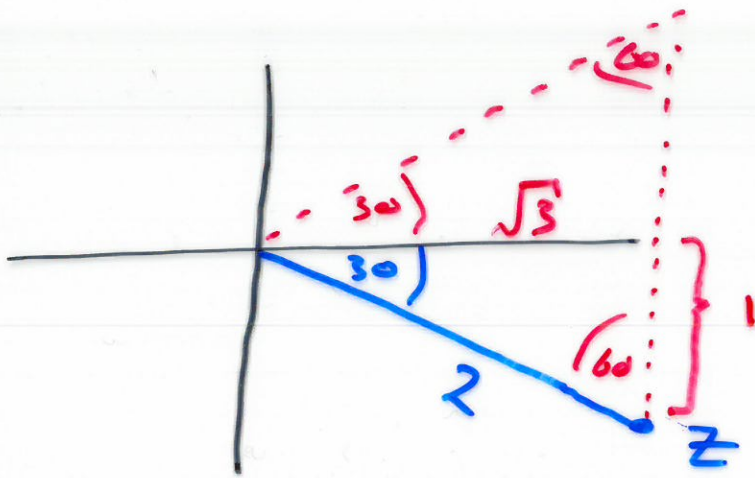
$$|z| = \left| \frac{v^7}{w^3} \right| = \frac{|v|^7}{|w|^3} = \frac{2^7}{4^3} = \frac{2^7}{2^6} = 2$$

$$\text{Arg}(z) = \text{Arg}\left(\frac{v^7}{w^3}\right) = \text{Arg}(v^7) - \text{Arg}(w^3)$$

$$= 7 \text{Arg}(v) - 3 \text{Arg}(w)$$

$$= 7 \cdot 15 - 3 \cdot 45$$

$$= 105 - 135 = -30^\circ$$



$$z = \sqrt{3} - i$$

$$4) \quad e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos \theta - i \sin \theta$$

$$e^{i\theta} + e^{-i\theta} = 2 \cos \theta$$

$$\text{Hence } \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$